आत्म–निर्भर भारत COMPLETE SELF-PREP BOOK

SAMPLE
QUESTION PAPERS
MATHEMATICS

CBSE CLASS 12

ALL PAPERS STRICTLY ON REDUCED SYLLABUS AND
AS PER LATEST CBSE SAMPLE PAPER PROVIDED ON 9th OCT 2020

WITH
TIME MANAGEMENT CHART
+
IMPORTANT Q's RELATED THEORY
+
2021 TOPPER TIP'S

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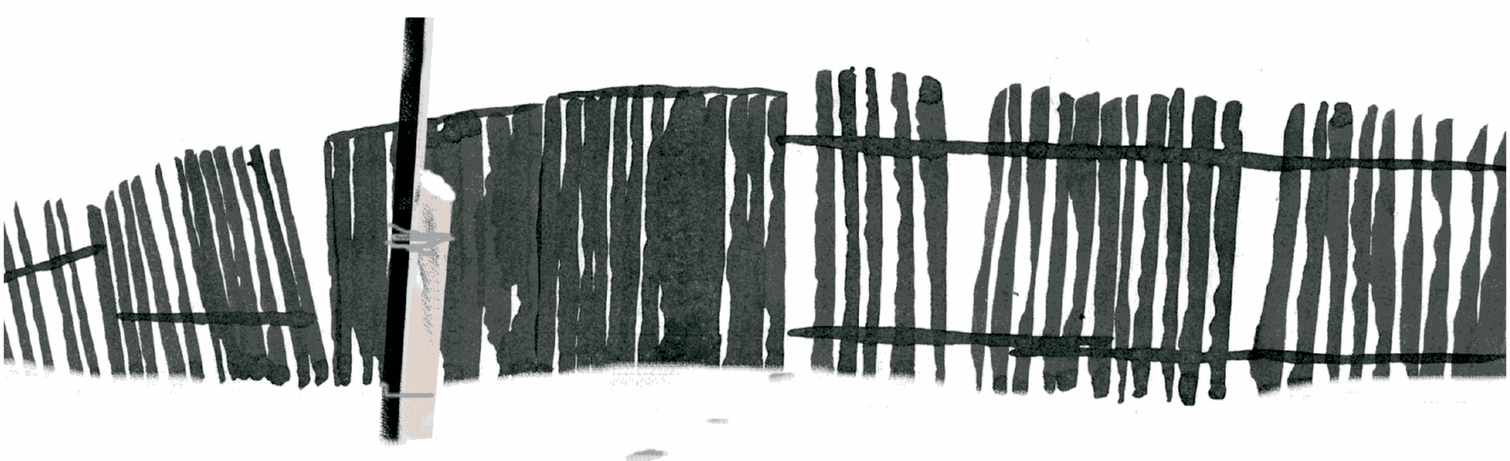


Hello!

Friends, this year is all about keeping caution, strengthening determination and smart learning. CBSE has made sweeping changes in the paper pattern of all subjects and we at Educart have adhered 100% to those changes.

After the record breaking sales and acceptance of our sample papers last year pan-india, we have launched class 12 books with some critical value additions. This is our special self-prep version for 2021 with the new objective section included.

EduCart has also roped in CBSE Mathematics experts and most experienced teachers, to analyse the new pattern and prepare a fine XIIth Class Mathematics Sample Papers Book for 2021.

Go break a leg!

Reviews

★★★★★ Read carefully what I am writing

By Sathya Raji on 2 October, 2020

“ Guys, this is a very emotional review who has gone through a lot. I lost confidence because of the lack of interest in studies. Dad said focus only on studies but I only like TikTok and PUBG. Now both got banned and I had no other option but to study effectively as mid-terms were near. Now 4 months has passed and I had no preparation of boards at all. So I decided to change things and bought EDUCART.

Their maps (mind maps rather) for the first time in life helped me understand that what all comes in the chapters & what's important in those chapters. I was actually being able to study. I mean how can someone put so much effort in writing the book. So that definately helped me figure some topics well. Today, I finished 2 chapters of chemistry from Educart book and managed to make my father proud.

★★★★★ Worth for 💰

By Malika on 3 September, 2020

“ It's a very good book for the candidates appearing in 2021.... very nice explanation and also very nice editing 👍👍 Go for itt!!!!!!

★★★★★ Wonderful book 😊

By S S on 10 September, 2020

“ Every paper has CBSE questions written in neat way with explanations and related theory. My father purchased this book for me as i m weak in science but i am so happy with it that im posting the review myself to thank educart personally. Edcart, please continue to make such books, in this covid time, this book is what we needed reallly!

T S Sudhir

(Author of Saina Nehwal's Biography | Journalist | Educator)

To: quickreply@agpgroup.in

“ Educart Exemplar is my suggested book for this year and I rarely recommend books. This one I have thoroughly read and liked for my students.

5★ Great product

Student

“ I recommend this book..... magnificent book for revision...so many good questions are there... My God! the mind maps are super cool... A must buy book ...for class 10 students...just a little mistakes are there but it doesn't matter as those are check points of your learning Mustbuy 👍👍👍👍👍

Dear Sir

6.77M Subscribers

“ India ki pehli atma-nirbhar self prep book that really no publisher can match with. Educart question bank is a must buy for all students!

RC Chauhan

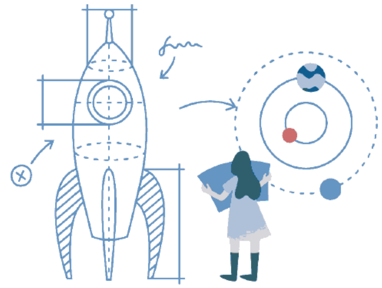
HOD of Mathematics - DPS

To: quickreply@agpgroup.in

“ We have reviewed countless Xth Class Maths books but Educart's Sample papers is our top recommendation. Educart has done their homework well on how CBSE students nowadays want to learn solving of maths standard questions.

Contents

** Self-assessment papers' solution are available on our website (www.educart.net).*



Question Paper Design

Section / Part	PART A		PART B		
	Objective type (VSA)	**Objective type** (Case-based)	**Subjective type** (Short Answer SA-1)	**Subjective type** (Short Answer SA-2)	**Subjective type** (Long Answer LA)
	1m each	*4m (1m per MCQ)*	*2m each*	*3m each*	*5m each*
Section I	16Q (5 choice)	–	–	–	–
Section II	–	2Q (4 MCQ each) (2 choice)	–	–	–
Section III	–	–	10Q (3 choice)	–	–
Section IV	–	–	–	7Q (2 choice)	–
Section V	–	–	–	–	3Q (3 choice)
Total Marks	**16m**	**8m**	**20m**	**21m**	**15m**

Note: This blueprint is prepared for simplicity purpose, based on the CBSE Sample Paper provided on 9th October 2020.

Time Management

The 3 hour long board paper needs to be attempted strategically so that you are not cut short for time on any question at the end. This means you need to complete each section of the paper within a pre-defined duration. Our experts have figured out the optimum time duration for each section for you to keep in mind. Please see below the time management chart for all subjects:

MATHEMATICS				
Part	**Section (Question Type)**	**Questions**	**Time To Be Spent (Per Question)**	**Total Time**
A	Section I (VSA)	16Q (1m each)	2 min per question	16 x 2 = **32 min**
	Section II (CBQ)	8Q (1m each)	3 min reading time x (2 passages) 2 min per question	(2 x 3) + (8 x 2) = **22 min**
B	Section III (SA–1)	10Q (2m each)	4 min per question	10 x 4 = **40 min**
	Section IV (SA–2)	7Q (3m each)	6 min per question	7 x 6 = **42 min**
	Section V (LA)	3Q (5m each)	8 min per question	3 x 8 = **24 min**
				Total Time: **2 hours 40 min**
				Revision Time: **20 min**

Topper Tips

(on cracking new pattern)

Friends, on request of the Educart team, followed below are some points I've prepared for you to keep in mind whilst attempting the Class 12 Board Exams on the new pattern:

Units Representation Should Be Correct

The following units should be written correctly always:

Length - cm, mm, m, km (not as cms, mms, ms, kms)

Area - sq cm, sq m, sq km (not as cm^2, mm^2, m^2, km^2)

Volume - cu cm, cu m, cu km (not as cm^3, mm^3, m3, km^3, etc)

Speed and Mass - km/h, kg, g (not as km/hr, kgs, gs)

Double-check numerical Values

Very common mistake we make is to not copy the correct values (numbers, equations etc) from the Question itself as we are in a rush. Ensure you read the questions word by word with great care.

Use Of Graph Paper

Graph paper should be used when necessary. Diagrams should be neatly drawn to score full marks.

Final Answer Marks

½ to 1 Marks are allocated just for the concluding answer. Ensure to mention the final answer neatly and correctly at the end of the solution. However, remember your working or method/steps contain majority marks.

Notes and Derivations driven practice (Physics)

- This year the Physics objective section has various types of conceptual questions from Assertion Reason to VSA to Case Based MCQs. Make a complete list of derivations, formulae and experiments in your syllabus and keep that list handy. Mind Maps given in this book will help in that.
- While solving a derivation, try and comprehend the logic behind the derivations and revise the concepts regularly. If you do not like the numerical part, start early! Get used to the numerical part. A Physics paper without a numerical is like a comb without teeth.
- Do not forget to mention the S.I units (if any) of all physical entities.

Select your MCQ options wisely (Hindi and English Core)

50% paper is now MCQs based. Do not rush into choosing a particular option. If unable to find the answer; use the rule of elimination to reach the most appropriate answer. Usually ruling out other 3 options works out faster.

Cracking Case-based Questions

Here is the trick. CBSE cannot ask any MCQ in case-based questions that is going to take you more than 1-2 minutes to solve as it will be a 1m MCQ each. So don't worry about the length of the question, treat it like a normal value input or understanding or remembering based question and move ahead. Time is of the essence.

New Pattern MCQs (English)

CBSE has made a complete overhaul of MCQ's style of questioning in Reading and Literature comprehensions of English Section A. They are not direct but inference based and analytical thinking driven. Educart has provided detailed explanations in this book for such MCQs to help understand how to come to a conclusive option.

15 minutes reading time hack

- There is 30-50% internal choice this time in each section. You get good 15 minutes in the beginning to read the question paper. Use this time to mark the choice questions you are more confident in attempting to avoid wasting critical thinking time while writing the exam.
- Mark the tough questions you definitely don't know the answer to or where you feel you will struggle, and remember to leave space to come back to answering them.

Prioritise your Sections order

Decide which Section you would want to attempt first and which Section at last. Always attempt the easy questions first. This way your confidence will grow and you will be mentally ready to take on the more challenging questions.

Answers Structure has to be right

- Write most of the answers in bullet points (with headings) or in a tabular form where possible to save time and stick to the point. CBSE paper checkers prefer such format to make it easy to allot full marks.
- Underline key (value) points for all answers and follow word count to save on time.
- Explain lengthy answers with examples and diagrams.
- Recheck for all logics and calculations in case of numerical.

Syllabus

(Reduced)

Units	Unit Name	Marks
I	Relations and Functions	08
II	Algebra	10
III	Calculus	35
IV	Vectors and Three - Dimensional Geometry	14
V	Linear Programming	05
VI	Probability	08
	Total	**80**
	Internal Assessment	**20**

Unit-I: Relations and Functions

1. Relations and Functions

Types of relations: reflexive, symmetric, transitive and equivalence relations. One to one and onto functions.

2. Inverse Trigonometric Functions

Definition, range, domain, principal value branch.

Unit-II: Algebra

1. Matrices

Concept, notation, order, equality, types of matrices, zero and identity matrix, transpose of a matrix, symmetric and skew symmetric matrices. Operation on matrices: Addition and multiplication and multiplication with a scalar. Simple properties of addition, multiplication and scalar multiplication. Noncommutativity of multiplication of matrices, Invertible matrices; (Here all matrices will have real entries).

2. Determinants

Determinant of a square matrix (up to 3 x 3 matrices), minors, co-factors and applications of determinants in finding the area of a triangle. Adjoint and inverse of a square matrix. solving system of linear equations in two or three variables (having unique solution) using inverse of a matrix.

Unit-III: Calculus

1. Continuity and Differentiability

Continuity and differentiability, derivative of composite functions, chain rule, derivative of inverse trigonometric functions, derivative of implicit functions. Concept of exponential and logarithmic functions.

Derivatives of logarithmic and exponential functions. Logarithmic differentiation, derivative of functions expressed in parametric forms. Second order derivatives.

2. Applications of Derivatives

Applications of derivatives: increasing/decreasing functions, tangents and normals, maxima and minima (first derivative test motivated geometrically and second derivative test given as a provable tool). Simple problems (that illustrate basic principles and understanding of the subject as well as reallife situations).

3. Integrals

Integration as inverse process of differentiation. Integration of a variety of functions by substitution, by partial fractions and by parts, Evaluation of simple integrals of the following types and problems based on them.

$$\int\frac{dx}{x^2\pm a^2}, \int\frac{dx}{\sqrt{x^2\pm a^2}}, \int\frac{dx}{\sqrt{a^2-x^2}}, \int\frac{dx}{ax^2+bx+c}, \int\frac{dx}{\sqrt{ax^{2+bx+c}}}$$

$$\int\frac{px+q}{ax^2+bx+c}dx, \int\frac{px+q}{\sqrt{ax^2+bx+c}}dx, \int\sqrt{a^2\pm x^2}dx, \int\sqrt{x^2-a^2}dx$$

Fundamental Theorem of Calculus (without proof).Basic properties of definite integrals and evaluation of definite integrals.

4. Applications of the Integrals

Applications in finding the area under simple curves, especially lines, parabolas; area of circles /ellipses (in standard form only) (the region should be clearly identifiable).

5. Applications of the Integrals

Definition, order and degree, general and particular solutions of a differential equation. Solution of differential equations by method of separation of variables, solutions of homogeneous differential equations of first order and first degree of the type: $\frac{dy}{dx} = f(y/x)$

Solutions of linear differential equation of the type:

$\frac{dy}{dx} + py = q$, where p and q are functions of x or constant.

Unit-IV: Vectors and Three-Dimensional Geometry

1. Vectors

Vectors and scalars, magnitude and direction of a vector. Direction cosines and direction ratios of a vector. Types of vectors (equal, unit, zero, parallel and collinear vectors), position vector of a point, negative of a vector, components of a vector, addition of vectors, multiplication of a vector by a scalar, position vector of a point dividing a line segment in a given ratio. Definition, Geometrical Interpretation, properties and application of scalar (dot) product of vectors, vector (cross) product of vectors.:

2. Three - dimensional Geometry

Direction cosines and direction ratios of a line joining two points. Cartesian equation and vector equation of a line, coplanar and skew lines, shortest distance between two lines. Cartesian and vector equation of a plane. Distance of a point from a plane.

Unit-V: Linear Programming

1. **Linear Programming**

   Introduction, related terminology such as constraints, objective function, optimization, different types of linear programming (L.P.) problems. graphical method of solution for problems in two variables, feasible and infeasible regions (bounded), feasible and infeasible solutions, optimal feasible solutions (up to three non-trivial constraints).

Unit-VI: Probability

1. **Probability**

   Conditional probability, multiplication theorem on probability, independent events, total probability, Bayes' theorem, Random variable and its probability distribution.

Deleted

(For 2021 Exam)

Unit-I: Relations and Functions

1. **Relations and Functions**

   Composite functions, inverse of a function.

2. **Inverse Trigonometric Functions**

   Graphs of inverse trigonometric functions

   Elementary properties of inverse trigonometric functions

Unit-II: Algebra

1. **Matrices**

   Existence of non-zero matrices whose product is the zero matrix.

   Concept of elementary row and column operations.

   Proof of the uniqueness of inverse, if it exists.

2. **Determinants**

   Properties of determinants

   Consistency, inconsistency and number of solutions of system of linear equations by examples

Unit-III: Calculus

1. **Continuity and Differentiability**

   Rolle's and Lagrange's Mean Value Theorems (without proof) and their geometric interpretation.

2. **Applications of Derivatives**

   Rate of change of bodies, use of derivatives in approximation

3. **Integrals**

   $\int\sqrt{ax^2+bx+c}\,dx$

   $\int(ax+b)\sqrt{ax^2+bx+c}\,dx$

   Definite integrals as a limit of a sum

4. Applications of the Integrals

Area between any of the two above said curves

5. Differential Equations

formation of differential equation whose general solution is given.

Solutions of linear differential equation of the type:

$\frac{dx}{dy} + px = q$, where p and q are functions of y or constants.

Unit-IV: Vectors and Three-Dimensional Geometry

1. Vectors

Scalar triple product of vectors.

2. Three - dimensional Geometry

Angle between (i) two lines, (ii) two planes, (iii) a line and a plane.

Unit-V: Linear Programming

1. Linear Programming

Mathematical formulation of L.P. problems

(unbounded).

Unit-VI: Probability

1. Probability

Mean and variance of random variable. Binomial probability distribution.

FAQs

1. Can we use black pen in CBSE board exam 2021?

As per last year's CBSE guideline, students appearing for CBSE Board Exams can write answers ONLY with a Blue color pen (blue or royal blue). It should be a ball point, gel or fountain pen.

If the students want to use a black pen to highlight or bold the points in answers or for writing titles or headlines then it is allowed.

2. Will the CBSE 2021 paper on reduced syllabus come based on the sample paper CBSE released? Will the difficulty level be the same?

Yes, it will be exactly as per the paper pattern and type of questions introduced by CBSE in the 9th October 2020 uploaded Sample paper. As far as the difficulty level is concerned, expect an easier paper than the provided sample paper as CBSE will not want to reduce chances of students to pass considering COVID-19 has made things a bit difficult. However, this Educart book is prepared keeping a medium difficulty level to prepare students fully for the upcoming new pattern paper.

3. When will CBSE provide datesheet for 2021 boards?

Exact dates for all subjects' exams is usually provided in the month of December of the ongoing academic session. Last year it came on 16th December 2019. Expect the same in the month of December and expect the exams start date to be later than March for the 2020-21 session.

4. How shall I prepare when there is not much time left?

When in shortage of time, less material to study from, is better. This can be done by focusing on only NCERT books (for theory) and our Educart sample papers for practice and nothing else. Educart Sample papers book is 100% designed on the upcoming 2021 paper to help you cover questions on all possible topics with detailed explanations.

5. What is the Pass Marks Cut-off and Criteria?

A candidate has to obtain a grade higher than E (i.e. atleast 33% marks) in all the five subjects of external examination in the main or at the compartmental examinations.

6. How do I access latest CBSE circulars and announcements?

You can always email us on quickreply@agpgroup.in for any update you want.
As far as official source is concerned, refer: www.cbseacademic.nic.in/circulars.html.

7. What is the process of applying for a recheck of Marks in a particular subject?

Any student has the right to do so within a week from date of declaration of CBSE board exam result.

The whole process of verification of marks is done online.Steps to apply for verification/rechecking of the answer sheet, are as follows: Apply for rechecking of marks on the CBSE's website www.cbse.nic.in by filling in your details and paying Rs. 500 per subject online (only). The result of verification of marks will be uploaded on the website automatically.

Overall, the verification will be restricted to checking whether all the answers have been checked, there has been no mistake in totalling of marks for each question and the marks have been transferred correctly on the title page of the answer book. A candidate may also apply for obtaining a copy of the evaluated answer book(s) at a later stage if not satisfied with the evaluation

8. What is the best way to practice from this book to score good marks?

In order to crack the board exam, this book is custom made to start with Topper Tips and Time management. This includes an explanation of how to smartly structure your 3 hours during the paper.

Once, you have covered the basics, you can go through the exclusive CBSE last year Topper hand-written solutions and CBSE papers to get a feel of what is normally asked and how to answer them.

Then you start with our most likely 6 solved sample papers, where you time yourself to complete each paper and cross-check your performance with our detailed solutions.

Lastly, the unsolved papers help you self-assess without the temptation of looking at the back and fine-tune your preparation. These are solid papers that, if done well will fully prepare you to do well in the 2021 board exam.

9. Who should I reach out to for any issue related to examination, re-evaluation of copy or any serious matter?

Ideally your only point of contact should be your school and they will take action on your behalf by submitting a request to CBSE regional office. However, we have managed to source some useful contacts in CBSE. Please refer to the next page for more information.

IMPORTANT CBSE CONTACTS

Lots of students and parents face the problem of not knowing how best to contact CBSE for matters related to *Examination, admission fees, last-minute change of subject, direct admissions, passing criteria, examination centre related issue, unfair means or even re-evaluation of results* if not satisfactory. This list is not exhaustive.

We have compiled a comprehensive list of contacts of your nearest CBSE Regional Offices for various issues depending on the region you belong to. CBSE prefers any request to be sent to Regional Offices only and that also via the head of your school ideally. It is, therefore, advised to make the request accordingly through a proper channel for prompt and timely action.

Your School Location/Region	**CBSE Regional Office (RO) Contact Details**
General	Dr. Sanyam Bhardwaj (Controller of Examinations) sanyamb.cbse@nic.in | 011-22515828 Dr. Joseph Emmanuel (Director (Academics) directoracad.cbse.nic.in | 011-23212603
Delhi, Foreign Schools	CBSE, PS-1-2, Institutional Area, I.P. Extn, Patparganj, Delhi - 110092 rodelhi.cbse@nic.in | 91-11-22239177-80, 22235948, 22235904
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CBSE
SAMPLE PAPER
9th October 2020

MATHEMATICS

Time Allowed: 3 Hours *Maximum Marks: 80*

General Instructions:

(i) This question paper contains **two parts A** and **B**. Each part is compulsory. Part A carries **24** marks and Part B carries **56** marks.

(ii) **Part-A** has Objective Type Questions and **Part-B** has Descriptive Type Questions.

(iii) Both Part A and Part B have choices.

PART-A

(i) It consists of two sections- **I** and **II**.

(ii) Section **I** comprises of 16 very short answer type questions.

(iii) Section **II** contains **2** case studies. Each case study comprises of 5 case-based MCQs. An examinee is to attempt **any 4 out of 5 MCQs**.

PART-B

(i) It consists of three sections- **III**, **IV** and **V**.

(ii) Section III comprises of 10 questions of **2 marks** each.

(iii) Section IV comprises of 7 questions of **3 marks** each.

(iv) Section V comprises of 3 questions of **5 marks** each.

(v) Internal choice is provided in **3** questions of Section –III, **2** questions of Section-IV and **3** questions of Section-V. You have to attempt only one of the alternatives in all such questions.

PART - A **24 marks**

SECTION - I

All questions are compulsory. In case of internal choices attempt any one.

1. Check whether the function $f : R \to R$ defined as $f(x) = x^3$ is one-one or not. 1

OR

How many reflexive relations are possible in a set A whose n (A) = 3. 1

2. A relation R in S = {1, 2, 3} is defined as R = {(1, 1), (1, 2), (2, 2), (3, 3)}. Which element(s) of relation R be removed to make R an equivalence relation? 1

3. A relation R in the set of real numbers R defined as R = {(*a*, *b*) : $\sqrt{a} = b$} is a function or not. Justify 1

OR

An equivalence relation R in A divides it into equivalence classes A_1, A_2, A_3. What is the value of $A_1 \cup A_2 \cup A_3$ and $A_1 \cap A_2 \cap A_3$. 1

4. If A and B are matrices of order 3 × *n* and *m* × 5 respectively, then find the order of matrix 5A – 3B, given that it is defined. 1

5. Find the value of A^2, where A is a 2 × 2 matrix whose elements are given by $a_{ij} = \begin{cases} 1 \text{ if } i = j \\ 0 \text{ if } i = j \end{cases}$ 1

OR

Given that A is a square matrix of order 3 × 3 and |A| = –4. Find |adj A|. 1

6. Let A = $[a_{ij}]$ be a square matrix of order 3 × 3 and |A| = –7. Find the value of 1

$a_{11} A_{21} + a_{12}A_{22} + a_{13}A_{23}$

where A_{ij} is the cofactor of element a_{ij}.

7. Find $\int e^x (1 - \cot x + \operatorname{cosec}^2 x)dx$ 1

OR

Evaluate $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x^2 \sin x \, dx$ 1

8. Find the area bounded by $y = x^2$, the *x*–axis and the lines $x = -1$ and $x = 1$. 1

9. How many arbitrary constants are there in the particular solution of the differential equation $\frac{dy}{dx} = -4xy^2;\ y(0) = 1$. 1

OR

For what value of n is the following a homogeneous differential equation: 1

$$\frac{dy}{dx} = \frac{x^3 - y^n}{x^2y + xy^2}$$

10. Find a unit vector in the direction opposite to $-\frac{3}{4}\hat{j}$. 1

11. Find the area of the triangle whose two sides are represented by the vectors $2\hat{i}$ and $-3\hat{j}$. 1

12. Find the angle between the unit vectors $\hat{a}$ and $\hat{b}$, given that $|\hat{a} + \hat{b}| = 1$. 1

13. Find the direction cosines of the normal to YZ plane? 1

14. Find the coordinates of the point where the line $\frac{x+3}{3} = \frac{y-1}{-1} = \frac{z-5}{-5}$ cuts the XY plane. 1

15. The probabilities of A and B solving a problem independently are $\frac{1}{3}$ and $\frac{1}{4}$ respectively. If both of them try to solve the problem independently, what is the probability that the problem is solved? 1

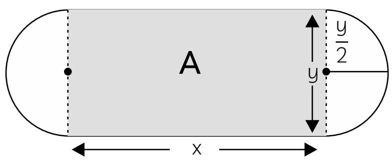
16. The probability that it will rain on any particular day is 50%. Find the probability that it rains only on first 4 days of the week. 1

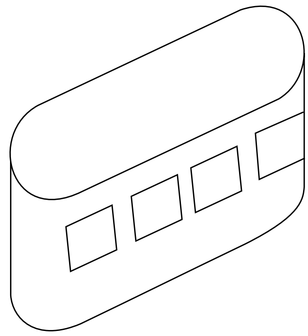
SECTION - II

Both the Case study based questions are compulsory. Attempt any 4 sub parts from each question. Each question carries 1 mark.

17. An architect designs a building for a multi-national company. The floor consists of a rectangular region with semicircular ends having a perimeter of 200m as shown below:

Design of Floor





Building

Based on the above information answer the following:

(i) If x and y represents the length and breadth of the rectangular region, then the relation between the variables is: 1

(a) $x + \pi y = 100$ (b) $2x + \pi y = 200$

(c) $\pi x + y = 50$ (d) $x + y = 100$

(ii) The area of the rectangular region A expressed as a function of x is: 1

(a) $\frac{2}{\pi}(100x - x^2)$ (b) $\frac{1}{\pi}(100x - x^2)$

(c) $\frac{x}{\pi}(100 - x)$ (d) $\pi y^2 + \frac{2}{\pi}(100x - x^2)$

(iii) The maximum value of area A is: 1

(a) $\frac{\pi}{3200}m^2$ (b) $\frac{3200}{\pi}m^2$

(c) $\frac{5000}{\pi}m^2$ (d) $\frac{1000}{\pi}m^2$

(iv) The CEO of the multi-national company is interested in maximizing the area of the whole floor including the semi-circular ends. For this to happen the valve of x should be: 1

(a) 0 m (b) 30 m

(c) 50 m (d) 80 m

(v) The extra area generated if the area of the whole floor is maximized is: 1

(a) $\frac{3000}{\pi}m^2$ (b) $\frac{5000}{\pi}m^2$

(c) $\frac{7000}{\pi}m^2$ (d) No change Both areas are equal

18. In an office three employees Vinay, Sonia and Iqbal process incoming copies of a certain form. Vinay process 50% of the forms. Sonia processes 20% and Iqbal the remaining 30% of the forms. Vinay has an error rate of 0.06, Sonia has an error rate of 0.04 and Iqbal has an error rate of 0.03

Based on the above information answer the following:

(i) The conditional probability that an error is committed in processing given that Sonia processed the form is: 1

(a) 0.0210 (b) 0.04
(c) 0.47 (d) 0.06

(ii) The probability that Sonia processed the form and committed an error is: 1

(a) 0.005 (b) 0.006
(c) 0.008 (d) 0.68

(iii) The total probability of committing an error in processing the form is: 1

(a) 0 (b) 0.047
(c) 0.234 (d) 1

(iv) The manager of the company wants to do a quality check. During inspection he selects a form at random from the days output of processed forms. If the form selected at random has an error, the probability that the form is NOT processed by Vinay is: 1

(a) 1 (b) 30/47
(c) 20/47 (d) 17/47

(v) Let A be the event of committing an error in processing the form and let E_1, E_2 and E_3 be the events that Vinay, Sonia and Iqbal processed the form. The value of $\Sigma_{i=1}^{3} P(E_i | A)$ is: 1

(a) 0 (b) 0.03
(c) 0.06 (d) 1

PART - B **56 marks**

SECTION - III

19. Express $\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right), \frac{-3\pi}{2}<x<\frac{\pi}{2}$ in the simplest form. 2

20. If A is a square matrix of order 3 such that $A^2 = 2A$, then find the value of $|A|$. 2

OR

If $A=\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, show that $A^2 - 5A + 7I = 0$. Hence find A^{-1}. 2

21. Find the value(s) of k so that the following function is continuous at $x = 0$ 2

$$f(x)=\begin{cases} \dfrac{1-\cos kx}{x\sin x} & if \;\; x \neq 0 \\ \dfrac{1}{2} & if \;\; x=0 \end{cases}$$

22. Find the equation of the normal to the curve

$y = x + \frac{1}{x},\ x > 0$ perpendicular to the line $3x - 4y = 7$. 2

23. Find $\int \frac{1}{\cos^2 x(1-\tan x)^2} dx$ 2

OR

Evaluate $\int_0^1 x(1-x)^n dx$ 2

24. Find the area of the region bounded by the parabola $y^2=8x$ and the line $x = 2$. 2

25. Solve the following differential equation: 2

$\frac{dy}{dx} = x^3 \operatorname{cosec} y$ given that $y(0) = 0$.

26. Find the area of the parallelogram whose one side and a diagonal are represented by coinitial vectors $\hat{i} - \hat{j} + \hat{k}$ and $4\hat{i} + 5\hat{k}$ respectively 2

27. Find the vector equation of the plane that passes through the point (1, 0, 0) and contains the line $\vec{r} = \lambda \hat{j}$. 2

28. A refrigerator box contains 2 milk chocolates and 4 dark chocolates. Two chocolates are drawn at random. Find the probability distribution of the number of milk chocolates. What is the most likely outcome?

OR

Given that E and F are events such that P(E) = 0.8, P(F) = 0.7, P (E∩F) = 0.6. Find $P(\bar{E} \mid \bar{F})$. 2

SECTION - IV

All questions are compulsory. In case of internal choices attempt any one.

29. Check whether the relation R in the set Z of integers defined as R = {(*a*, *b*) : *a* + *b* is "divisible by 2"} is reflexive, symmetric or transitive. Write the equivalence class containing 0 i.e. [0]. 3

30. If $y = e^{x\sin^2 x} + (\sin x)^x$, find $\frac{dy}{dx}$. 3

31. Prove that the greatest integer function defined by $f(x) = [x]$, $0 < x < 2$ is not differentiable at $x = 1$. 3

OR

If $x = a \sec\theta$, $y = b\tan\theta$ find $\frac{d^2y}{dx^2}$ at $x = \frac{\pi}{6}$. 3

32. Find the intervals in which the function *f* given by

$f(x) = \tan x - 4x,\ \ x \in \left(0, \frac{\pi}{2}\right)$ is

(a) strictly increasing

(b) strictly decreasing 3

33. Find $\int \frac{x^2+1}{(x^2+2)(x^2+3)} dx$. 3

34. Find the area of the region bounded by the curves $x^2 + y^2 = 4$, $y = \sqrt{3}x$ and *x-axis* in the first quadrant. 3

OR

Find the area of the ellipse $x^2 + 9y^2 = 36$ using integration. 3

35. Find the general solution of the following differential equation: $x\,dy - (y + 2x^2)dx = 0$. 3

SECTION - V

All questions are compulsory. In case of internal choices attempt any one.

36. If $A = \begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$, find A^{-1}. Hence

Solve the system of equations;

$x - 2y = 10$

$2x - y - z = 8$

$-2y + z = 7$ 5

OR

Evaluate the product AB, where

$A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$

Hence solve the system of linear equations

$x - y = 3$

$2x + 3y + 4z = 17$

$y + 2z = 7$. 5

37. Find the shortest distance between the lines

$\vec{r} = 3\hat{i} + 2\hat{j} - 4\hat{k} + \lambda(\hat{i} + 2\hat{j} + 2\hat{k})$

and $\vec{r} = 5\hat{i} - 2\hat{j} + \mu(3\hat{i} + 2\hat{j} + 6\hat{k})$

If the lines intersect find their point of intersection. 5

OR

Find the foot of the perpendicular drawn from the point $(-1, 3, -6)$ to the plane $2x + y - 2z + 5 = 0$. Also find the equation and length of the perpendicular. 5

38. Solve the following linear programming problem (L.P.P) graphically.

Maximize $Z = x + 2y$

subject to constraints;

$x + 2y \geq 100$

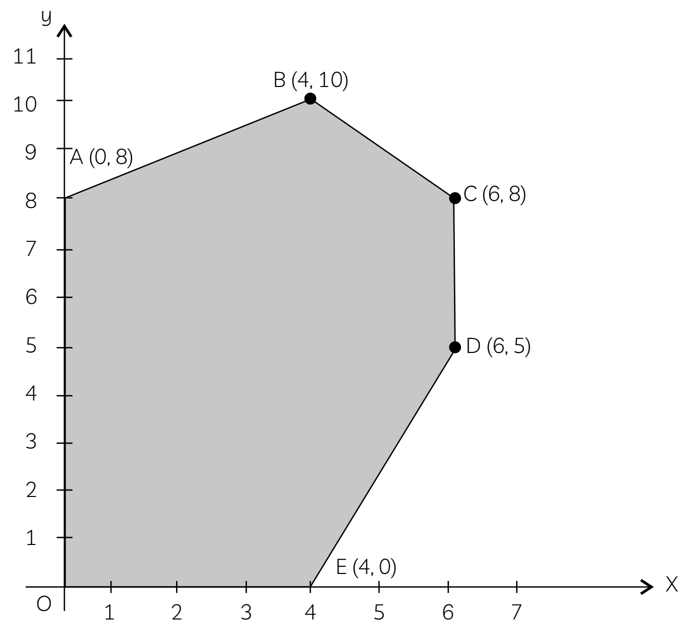
$2x - y \leq 0$

$2x + y \leq 200$

$x, y \geq 0$

OR

The corner points of the feasible region determined by the system of linear constraints are as shown below:



Answer each of the following:

(i) Let Z = $3x - 4y$ be the objective function. Find the maximum and minimum value of Z and also the corresponding points at which the maximum and minimum value occurs.

(ii) Let Z = $px + qy$, where $p, q > o$ be the objective function. Find the condition on p and q so that the maximum value of Z occurs at B (4, 10) and C (6, 8). Also mention the number of optimal solutions in this case. 5

1

SOLUTION

WITH CBSE MARKING SCHEME

Note: The text in all grey boxes are solutions given in the CBSE Marking Scheme 2020-2021.

PART - A

SECTION - I

1. Let $f(x_1) = f(x_2)$ for some $x_1, x_2 \in R$

$\Rightarrow (x_1)^3 = (x_2)^3$

$\Rightarrow x_1 = x_2$, Hence, $f(x)$ is one-one

Explanation: Let $f(x_1) = f(x_2)$ for some $x_2, x_2 \in R$.

Then, $x_1^3 = x_2^3$

$\Rightarrow x_1 = x_2$,

Hence, $f(x)$ is one-one

OR

2^6 reflexive relations

Explanation: Number of Reflexive relations on a set

$$= 2^{n^2 - n}$$

Here, $n = 3$

$\therefore$ Number of reflexive relations

$$= 2^{3^2 - 3}$$
$$= 2^{9-3} = 2^6$$

2. (1, 2)

Explanation: An equivalence relation is one that is reflexive, symmetric and transitive.

For reflexive, $(a, a) \in R$

if $(a, b) \in R$, then (b, a)

For symmetric, *if* $(a, b) \in R$, *then* (b, a)

For transitive, *if* $(a, b) \in R$, $(b, c) \in R$, *then* (c, a)

Then, (1, 2) should be removed to make the given relation an equivalence relation.

3. Since $\sqrt{a}$ is not defined for $a \in (-\infty, 0)$

$\therefore \sqrt{a} = b$ is not a function.

Explanation: Relation R is given as, R = $\{(a, b) : \sqrt{a} = b\}$.

Since, $\sqrt{a}$ is not defined for $a \in (-\infty, 0)$

$\therefore \sqrt{a} = b$ is not a function.

OR

$A_1 \cup A_2 \cup A_3 = A$ and $A_1 \cap A_2 \cap A_3 = \phi$

Explanation: Since, R is equivalence relation in A, which is divided into equivalence classes A_1, A_2, A_3.

Then, $A_1 \cup A_2 \cup A_3 = A$

(since, it is divided into 3 classes)

and $A_1 \cap A_2 \cap A_3 = \phi$

(as they have nothing in common)

4. 3×5

Explanation: 3×5

If 5A – 3B is defined, then order, of A and B matrices will be same.

Then 5A – 3B will be of the same order as A and B, which is 3×5.

5. $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \Rightarrow A^2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Explanation: Here,

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad \begin{matrix} 1 \text{ if } i \neq j \\ 0 \text{ if } i = j \end{matrix}$$

Then $A^2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

OR

$|\text{adj } A| = (-4)^{3-1} = 16$

Explanation: Order of square matrix,

$A = 3 \times 3$

Determinant of $|A| = -4$

Then, $|\text{adj } A| = |A|^{n-1}$

$= (-4)^{3-1} = (-4)^2 = 16$

6. 0

Explanation: "If elements of one row (or column) are multiplied by the cofactors of elements of any other row (or column) then their sum is zero."

7. $e^x (1 - \cot x) + C$

Explanation:

$$I = \int e^x (1 - \cot x + \text{cosec}^2 x) dx$$

$$= \int e^x dx - \int e^x \cot x \, dx + \int e^x \text{cosec}^2 x \, dx$$

$$= e^x - \left[\cot x \int e^x dx - \int \frac{d}{dx}(\cot x) e^x \, dx\right] + \int e^x \text{cosec}^2 x \, dx$$

$$= e^x - e^x \cot x - ® \, e^x \text{cosec}^2 x \, dx + \int e^x \text{cosec}^2 x \, dx + c$$

$$= e^x (1 - \cot x) + c$$

OR

$\because f(x)$ is an odd function

$$\therefore \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x^2 \sin x \, dx = 0$$

Explanation: Given, $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x^2 \sin x \, dx$

Let $I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x^2 \sin x \, dx$

Since, $\int_{-a}^{a} f(x) dx$

$$\begin{cases} 0, & \text{if } f(-x) = -f(x) \\ 2\int_0^a f(x) dx, & \text{if } f(-x) = f(x) \end{cases}$$

$f(x) = x^2 \sin x$

Then, $f(-x) = (-x)^2 \sin(-x)$

$= -x^2 \sin x = -f(x)$

$\therefore f(x)$ is an odd function.

Therefore, $I = 0$

8. $$A = 2\int_0^1 x^2 dx = \frac{2}{3}[x^3]_0^1$$

$$= \frac{2}{3} \text{ sq. unit}$$

Explanation: Here, $y = x^2$

Therefore, area, $A = ≡ y\,dx$

$$= \int_{-1}^{1} x^2 dx$$

$$= 2 ≡_0^1 x^2 dx \quad [\because f(-x) = f(x)]$$

$$= 2 \times \left[\frac{x^3}{3}\right]_0^1$$

$$= 2 \times \frac{1}{3} = \frac{2}{3} \text{ sq. units}$$

9. 0

Explanation:

Since, according to the definition of particular solution of a differential equation, it does not contain any arbitary constants.

So, number of arbitary constants in a particular solution of a differential equation is '0'.

OR

3

Explanation:

In homogeneous differential equation the degree of terms are same. So value of n is 3.

10. $\hat{j}$

Explanation: A unit vector in direction opposite to $\frac{-3}{4}\hat{j}$ will be in positive y-axis.

$\therefore$ Required vector is $\hat{j}$

11. $\frac{1}{2}|2\hat{i} \times (-3\hat{j})| = \frac{1}{2}|-6\hat{k}| = 3$ sq. units

Explanation: Area of triangle

$$= \frac{1}{2}|\vec{a} \times \vec{b}|$$

$$= \frac{1}{2}|2\hat{i} \times (-3\hat{j})|$$

$$= \frac{1}{2}|-6\hat{k}| \qquad [\because \hat{i} \times \hat{j} = \hat{k}]$$

$$= \frac{1}{2} \times 6 = 3 \text{ sq. units}$$

12. $|\hat{a} + \hat{b}|^2 = 1$

$\Rightarrow \hat{a}^2 + \hat{b}^2 + 2\hat{a}.\hat{b} = 1$

$\Rightarrow 2\hat{a}.\hat{b} = 1 - 1 - 1$

$\Rightarrow \hat{a}.\hat{b} = \frac{-1}{2}$

$\Rightarrow |\hat{a}||\hat{b}| \cos\theta = \frac{-1}{2}$

$\Rightarrow \theta = \pi - \frac{\pi}{3}$

$\Rightarrow \theta = \frac{2\neq}{3}$

Explanation: Given, $\hat{a}$ and $\hat{b}$ are unit vectors

and $|\hat{a} + \hat{b}| = 1$ ...(1)

On squaring both sides of equation (1), we get

$$(\hat{a} + \hat{b})^2 = 1$$

$\Rightarrow \hat{a}^2 + \hat{b}^2 + 2\hat{a}.\hat{b} = 1$

$\Rightarrow 2\hat{a}.\hat{b} = 1 - 1 - 1 \qquad [\because |\hat{a}| = |\hat{b}| = 1]$

$\Rightarrow 2\hat{a}.\hat{b} = -1$

$\Rightarrow \hat{a}.\hat{b} = \frac{-1}{2}$

$\Rightarrow |\hat{a}||\hat{b}|\cos\theta = \frac{-1}{2}$

$\Rightarrow \cos\theta = \frac{-1}{2}$

$\Rightarrow \theta = \cos^{-1}\left(\frac{-1}{2}\right)$

$= \cos^{-1}\left[\cos\left(\pi - \frac{\pi}{3}\right)\right]$

$\Rightarrow \theta = \frac{2\neq}{3}$

13. 1, 0, 0

Explanation: yz plane $\rightarrow x$ axis.

Then, direction ratios of normal to yz plane $= (x, 0, 0)$

Then

Direction Cosines

$$= \left[\frac{x}{\sqrt{x^2 + 0^2 + 0^2}}, \frac{0}{\sqrt{x^2 + 0^2 + 0^2}}, \frac{0}{\sqrt{x^2 + 0^2 + 0^2}}\right]$$

$= (1, 0, 0)$

14. (0, 0, 0)

Explanation: Coordinates of a point on XY-plane are $(a, b, 0)$.

Then, the point where the line and plane intersects, will satisfy the equation of line.

Then, $\frac{a+3}{3} = \frac{b-1}{-1} = \frac{0-5}{-5}$

$\Rightarrow \frac{a+3}{3} = 1, \frac{b-1}{-1} = 1$

$\Rightarrow a = 0, b = 0$

$\therefore$ coordinates of required point on XY plane are (0, 0, 0).

15. $1 - \frac{2}{3} \times \frac{3}{4} = \frac{1}{2}$

Explanation: $P(A) = \frac{1}{3}$ and $P(B) = \frac{1}{4}$

Probability that problem is solved

= Probability that A solves the problem or B solves the problem

$= P(A \cup B)$

$= P(A) + P(B) - P(A \cap B)$

Since, A and B are independent

$$P(A \cap B) = P(A)\,P(B)$$

$$= \frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$$

$$\therefore \quad P(A \cup B) = \frac{1}{3} + \frac{1}{4} - \frac{1}{12} = \frac{4+3-1}{12}$$

$$= \frac{6}{12} = \frac{1}{2}$$

Hence, required probability is $\frac{1}{2}$

16. $\left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^3 = \left(\frac{1}{2}\right)^7$

Explanation: Here,
Probability of rain,

$$p = \frac{50}{100} = \frac{1}{2}$$

Probability of no rain, $q = 1 - \frac{50}{100} = \frac{1}{2}$

Then, required probability $= \left(\frac{1}{2}\right)^4 \times \left(\frac{1}{2}\right)^3$

$$= \left(\frac{1}{2}\right)^7$$

SECTION - II

17. (i) (b) $2x + \pi y = 200$

Explanation: Since, perimeter of floor = 200 m

$$\therefore x + x + 2 \times \pi \left(\frac{y}{2}\right) = 200$$

$$\Rightarrow \quad 2x + \pi y = 200$$

(ii) (a) $\frac{2}{\pi}(100x - x^2)$

Explanation: Area of rectangular region,

$$A = xy$$

$$= x\left(\frac{200 - 2x}{\pi}\right)$$

(from equation obtained in Q-1)

$$= \frac{2}{\pi}(100x - x^2)$$

(iii) (c) $\frac{5000}{\neq}$ m^2

Explanation: Now

$$A = \frac{2}{\neq}(100x - x^2)$$

Then $\frac{dA}{dx} = \frac{2}{\neq}(100 - 2x)$

For maximum area

Put $\frac{dA}{dx} = 0$

$$\Rightarrow \frac{2}{\neq}(100 - 2x) = 0$$

$$\Rightarrow \quad x = 50$$

Since, $\frac{d^2A}{dx^2} = \frac{2}{p}(-x)$ *i.e.*, negative

$\therefore$ Maximum area $= \frac{2}{\neq}(5000 - 2500)$

$$= \frac{5000}{\neq} \text{ m}^2$$

(iv) (a) 0 m

Explanation: Area of complete floor

$$= xy + \pi\left(\frac{y}{2}\right)^2$$

$$A = xy + \frac{\pi y^2}{4}$$

$$A = \frac{2}{\pi}(100x - x^2) + \frac{\pi}{4}\left(\frac{200 - 2x}{\pi}\right)^2$$

$$A = \frac{2}{\pi}(100x - x^2) + \frac{1}{4\pi}(200 - 2x)^2$$

Now, $\frac{dA}{dx} =$

$$\frac{2}{\pi}(100 - 2x) + \frac{2}{4\pi}(200 - 2x) \times -2$$

For maximising area, put

$$\frac{dA}{dx} = 0$$

$$\frac{2}{\pi}(100 - 2x) = \frac{4}{4\pi}(200 - 2x)$$

$$\Rightarrow \quad 200 - 4x = 200 - 2x$$

$$\Rightarrow \quad -2x = 0$$

$$\Rightarrow \quad x = 0$$

(v) (d) No changes both area are equal

Explanation: Since, $x = 0$. So no change in area.

18. (i) (b) 0.04

Explanation:

E_1 = Vinay processed the form

E_2 = Sonia processed the form

E_3 = Iqbal processed the form

$$P(E_1) = \frac{50}{100} = \frac{1}{2}$$

$$P(E_2) = \frac{20}{100} = \frac{1}{5}$$

$$P(E_3) = \frac{30}{100} = \frac{3}{10}$$

Let A is an error committed.

$$P\left(\frac{A}{E_1}\right) = 0.06,\ P\left(\frac{A}{E_2}\right) = 0.04.\ P\left(\frac{A}{E_3}\right) = 0.03$$

$$P\left(\frac{A}{E_2}\right) = 0.04$$

(ii) (c)

$$P(\text{error}) = P(E_2) \times P\left(\frac{A}{E_2}\right)$$

$$= \frac{1}{5}\ 0.04 = 0.2 \times 0.04$$

$$= 0.008$$

(iii) (b)

Total probability of commiting error

$$P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right) + P(E_3)P\left(\frac{A}{E_3}\right)$$

$$= \frac{1}{2} \times 0.06 + \frac{1}{5} \times 0.04 + \frac{3}{10} \times 0.03$$

$$= 0.03 + 0.008 + 0.009$$

$$= 0.047$$

(iv) (d)

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)\,P\left(\frac{A}{E_1}\right)}{P(E_1)\,P\left(\frac{A}{E_1}\right) + P(E_2)\,P\left(\frac{A}{E_2}\right) + P(E_3)\,P\left(\frac{A}{E_3}\right)}$$

$$= \frac{\frac{1}{2} \times 0.06}{\frac{1}{2} \times 0.06 + \frac{1}{5} \times 0.04 + \frac{3}{10} \times 0.03}$$

$$= \frac{0.03}{0.047} = \frac{30}{47}$$

$\therefore$ Probability that form is not processed by Vinay

$$= 1 - P\left(\frac{E_1}{A}\right)$$

$$= 1 - \frac{30}{47}$$

$$= \frac{17}{47}$$

(v) (d)

$$= P\left(\frac{E_1}{A}\right) + P\left(\frac{E_2}{A}\right) + P\left(\frac{E_3}{A}\right)$$

$$= \frac{P(E_1)P\left(\frac{A}{E_1}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right) + P(E_1)P\left(\frac{A}{E_3}\right)}$$

$$+ \frac{P(E_2)\,P\left(\frac{A}{E_2}\right)}{P(E_1)\,P\left(\frac{A}{E_1}\right) + P(E_2)\,P\left(\frac{A}{E_2}\right) + P(E_3)\,P\left(\frac{A}{E_3}\right)}$$

$$+ \frac{P(E_3)\,P\left(\frac{A}{E_3}\right)}{P(E_1)\,P\left(\frac{A}{E_1}\right) + P(E_2)\,P\left(\frac{A}{E_2}\right) + P(E_3)\,P\left(\frac{A}{E_3}\right)}$$

$$= \frac{P(E_1)\,P\left(\frac{A}{E_1}\right) + P(E_2)\,P\left(\frac{A}{E_2}\right) + P(E_3)\,P\left(\frac{A}{E_3}\right)}{P(E_1)\,P\left(\frac{A}{E_1}\right) + P(E_2)\,P\left(\frac{A}{E_2}\right) + P(E_3)\,P\left(\frac{A}{E_3}\right)}$$

$$= 1$$

PART - B

SECTION - III

19. $\tan^{-1}\left(\dfrac{\cos x}{1 - \sin x}\right)$

$$= \tan^{-1}\left[\frac{\sin\left(\frac{\pi}{2} - x\right)}{1 - \cos\left(\frac{\pi}{2} - x\right)}\right]$$

$$\tan^{-1}\left[\frac{2\sin\left(\frac{\pi}{4} - \frac{x}{2}\right)\cos\left(\frac{\pi}{4} - \frac{x}{2}\right)}{2\sin^2\left(\frac{\pi}{4} - \frac{x}{2}\right)}\right]$$

$$\tan^{-1}\left[\cot\left(\frac{\pi}{4} - \frac{x}{2}\right)\right]$$

$$= \tan^{-1}\left[\tan^{-1}\frac{\pi}{2} - \left(\frac{\pi}{4} - \frac{x}{2}\right)\right]$$

$$\tan^{-1}\left[\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right] = \frac{\pi}{4} + \frac{x}{2}$$

Explanation: Given, $\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right)$

$$= \tan^{-1}\left(\frac{\sin\left(\frac{\pi}{2} - x\right)}{1 - \cos\left(\frac{\pi}{2} - x\right)}\right)$$

$$= \tan^{-1}\left[\frac{2\sin\left(\frac{\pi}{4} - \frac{x}{2}\right)\cos\left(\frac{\pi}{4} - \frac{x}{2}\right)}{2\sin^2\left(\frac{\pi}{4} - \frac{x}{2}\right)}\right]$$

$$= \tan^{-1}\left[\frac{\cos\left(\frac{\pi}{4} - \frac{x}{2}\right)}{\sin\left(\frac{\pi}{4} - \frac{x}{2}\right)}\right]$$

$$= \tan^{-1}\left[\cot\left(\frac{\pi}{4} - \frac{x}{2}\right)\right]$$

$$= \tan^{-1}\left[\tan\left\{\frac{\pi}{2} - \left(\frac{\pi}{4} - \frac{x}{2}\right)\right\}\right]$$

$$= \tan^{-1}\left[\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right] = \frac{\pi}{4} + \frac{x}{2}$$

20.

$$A^2 = 2A$$

$\Rightarrow$ $|AA| = |2A|$

$\Rightarrow$ $|A||A| = 8|A|$

$(\because |AB| = |A||B| \text{ and } |2A| = 2^3|A|)$

$\Rightarrow$ $|A|(|A| - 8) = 0$

$\Rightarrow$ $|A| = 0 \text{ or } 8$

Explanation: A is a square matrix of order 3. So, $n = 3$.

Also, $A^2 = 2A$ (Given)

$\Rightarrow$ $|AA| = |2A|$

$\Rightarrow$ $|A||A| = 8|A|$

$(\because |AB| = |A||B| \text{ and } |2A| = 2^n|A|)$

$\Rightarrow$ $|A|(|A| - 8) = 0$

$\Rightarrow$ $|A| = 0 \text{ or } 8$

OR

$$A^2 = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$$

$$5A = \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix}, 7I = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$\Rightarrow A^2 - 5A + 7I = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$$

$\Rightarrow$ $A^{-1}(A^2 - 5A + 7I) = A^{-1}0$

$\Rightarrow$ $A - 5I + 7A^{-1} = 0$

$\Rightarrow$ $7A^{-1} = 5I - A$

$$\Rightarrow A^{-1} = \frac{1}{7}\left(\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\right)$$

$$\Rightarrow A^{-1} = \frac{1}{7}\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$$

Explanation: here,

$$A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

Then, $A^2 = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

$$= \begin{bmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$$

$$5A = 5\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix}$$

$$7I = 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

Then, $A^2 - 5A + 7I$

$$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 8-15+7 & 5-5+0 \\ -5+5+0 & 3-10+7 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$$

Hence proved.

Now, $A^2 - 5A + 7I = 0$

Then, $A^{-1}(A^2 - 5A + 7I) = 0$

$\Rightarrow$ $A - 5I + 7A^{-1} = 0$

$\Rightarrow$ $7A^{-1} = 5I - A$

$$\Rightarrow A^{-1} = \frac{1}{7}(5I - A)$$

$$\Rightarrow A^{-1} = \frac{1}{7}\left(\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\right)$$

$$= \frac{1}{7}\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$$

Hence, the value of A^{-1} is $\frac{1}{7}\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$

21. $$\underset{x \to 0}{Lt} \frac{1 - \cos kx}{x \sin x}$$

$$= \underset{x \to 0}{Lt} \frac{2\sin^2\left(\frac{kx}{2}\right)}{x \sin x}$$

$$= \underset{x \to 0}{Lt} \frac{\frac{2\sin^2\left(\frac{kx}{2}\right)}{x^2}}{\frac{x \sin x}{x^2}}$$

$$= \frac{\underset{x \to 0}{Lt} \frac{2\sin^2\left(\frac{kx}{2}\right)}{\left(\frac{kx}{2}\right)^2} \times \left(\frac{k}{2}\right)^2}{\underset{x \to 0}{Lt} \frac{\sin x}{x}}$$

$$= \frac{2 \times 1 \times \frac{k^2}{4}}{1}$$

$\because$ $f(x)$ is continuous at $x = 0$

$\therefore \quad \underset{x \to 0}{Lt} f(x) = f(0)$

$\Rightarrow \quad \frac{k^2}{2} = \frac{1}{2} \Rightarrow k^2 = 1 \Rightarrow k = \pm 1$

Explanation:

$$\lim_{x \to 0} \frac{1 - \cos kx}{x \sin x} = \lim_{x \to 0} \frac{2 \sin^2\left(\frac{kx}{2}\right)}{x \sin x}$$

$$= \lim_{x \to 0} \frac{\frac{2 \sin^2\left(\frac{kx}{2}\right)}{x^2}}{\frac{x \sin x}{x^2}}$$

$$= \frac{2 \times \left[\lim_{x \to 0} \frac{\sin\left(\frac{kx}{2}\right)}{\left(\frac{kx}{2}\right)}\right]^2 \times \left(\frac{k}{2}\right)^2}{\lim_{x \to 0} \frac{\sin x}{x}}$$

$$= \frac{2 \times 1 \times \frac{k^2}{4}}{1} \qquad \left[\because \lim_{x \to 0} \frac{\sin x}{x} = 1\right]$$

$$= \frac{k^2}{2}$$

Since, $f(x)$ is continuous at $x = 0$

$\therefore \quad \lim_{x \to 0} f(x) = f(0)$

$\Rightarrow \quad \frac{k^2}{2} = \frac{1}{2} \Rightarrow k^2 = 1$

$\Rightarrow \quad k = \pm 1$

Hence, the value of k is 1 or – 1.

22. $$y = x + \frac{1}{x} \Rightarrow \frac{dy}{dx} = 1 - \frac{1}{x^2}$$

$\because$ normal is perpendicular to $3x - 4y = 7$,

$\therefore$ tangent is parallel to it

$$1 - \frac{1}{x^2} = \frac{3}{4} \Rightarrow x^2 = 4$$

$\Rightarrow \quad x = 2 \qquad (\because x > 0)$

when $x = 2$, $y = 2 + \frac{1}{2} = \frac{5}{2}$

$\therefore$ Equation of Normal :

$y - \frac{5}{2} = -\frac{4}{3}(x - 2) \Rightarrow 8x + 6y = 31$

Explanation: Given, equation of curve is

$$y = x + \frac{1}{x}$$

On differentiating both sides, we get

$$\frac{dy}{dx} = 1 - \frac{1}{x^2}$$

Since, normal is perpendicular to $3x - 4y = 7$

Then, tangent to this line is parallel to the normal

Now, Slope of line $3x - 4y = 7$ is,

$$\frac{dy}{dx} = \frac{3}{4}$$

$\therefore \quad 1 - \frac{1}{x^2} = \frac{3}{4}$

$x^2 = 4 \Rightarrow x = 2 \ (\because x > 0)$

Now, when $x = 2$

$$y = 2 + \frac{1}{2} = \frac{5}{2}$$

$\therefore$ Equation of normal is

$$\left(y - \frac{5}{2}\right) = -\frac{1}{\frac{dy}{dx}}(x - 2)$$

$$\Rightarrow \quad y - \frac{5}{2} - \frac{1}{\frac{3}{4}}(x - 2)$$

$$\Rightarrow \quad \frac{2y - 5}{2} = \frac{-4}{3}(x - 2)$$

$$\Rightarrow \quad 6y - 15 = -8x + 16$$

$$\Rightarrow \quad 8x + 6y = 31$$

23. $\mathrm{I} = \int \frac{1}{\cos^2 x(1 - \tan x)^2}dx$

Put, $1 - \tan x = y$

So that,

$$-\sec^2 x\, dx = dy$$

$$= \int \frac{-1dy}{y^2} = -\int y^{-2}dy$$

$$= +\frac{1}{y} + c = \frac{1}{1 - \tan x} + c$$

Explanation: Let,

$$\mathrm{I} = \int \frac{1}{\cos^2 x\,(1 - \tan x)^2}dx$$

Let $\quad 1 - \tan x = y$

Then,$-\sec^2$ x $dx = dy$

$$\therefore \qquad \mathrm{I} = \int \frac{-dy}{\sec^2 x \cos^2 x (y)^2}$$

$$= -\int \frac{dy}{y^2}$$

$$= -\int y^{-2}\, dy$$

$$= -\frac{y^{-2+1}}{-2+1} + c$$

$$= \frac{1}{y} + c$$

$$= \frac{1}{1 - \tan x} + c$$

where, c is the constant of integration.

OR

$$\mathrm{I} = \int_0^1 x(1 - x)^n dx$$

$$\mathrm{I} = \int_0^1 (1 - x)[1 - (1 - x)]^n dx$$

$$\mathrm{I} = \int_0^1 (1 - x)x^n dx = \int_0^1 (x^n - x^{n+1})dx$$

$$\mathrm{I} = \left[\frac{x^{n+1}}{n+1} - \frac{x^{n+2}}{n+2}\right]_0^1$$

$$\mathrm{I} = \left[\left(\frac{1}{n+1} - \frac{1}{n+2}\right) - 0\right]$$

$$= \frac{1}{(n+1)(n+2)}$$

Explanation: Let,

$$\mathrm{I} = \int_0^1 x(1 - x)^n\, dx$$

$$= \int_0^1 (1 - x)[1 - (1 - x)]^n\, dx$$

$$\left[\because \int_b^a f(x)dx = \int_b^a f(a + b - x)dx\right]$$

$$= \int_0^1 (1 - x)x^n\, dx$$

$$= \int_0^1 (x^n - x^{n+1})\, dx$$

$$\mathrm{I} = \left[\frac{x^{n+1}}{n+1} - \frac{x^{n+2}}{n+2}\right]_0^1$$

$$= \left[\left(\frac{1}{n+1} - \frac{1}{n+2}\right) - 0\right]$$

$$= \left[\frac{(n+2) - n - 1}{(n+1)(n+2)}\right]$$

$$= \frac{1}{(n+1)(n+2)}$$

24. $$\text{Area} = 2\int_0^2 \sqrt{8x}\, dx$$

$$= 2 \times 2\sqrt{2}\int_0^2 x^{\frac{1}{2}} dx$$

$$= 4\sqrt{2}\left[\frac{2}{3}x^{\frac{3}{2}}\right]_0^2$$

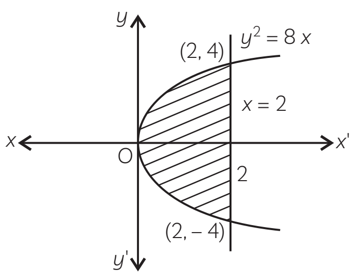
$$= \frac{8}{3}\sqrt{2}\left[2^{\frac{3}{2}} - 0\right]$$

$$= \frac{8\sqrt{2}}{3} \times 2\sqrt{2}$$

$$= \frac{32}{3} \text{ sq. units}$$

Explanation:

$$\text{Area} = 2\int_0^2 y\,dx$$



$$= 2\int_0^2 \sqrt{8x}\,dx$$

$$= 2 \times 2\sqrt{2}\int_0^2 \sqrt{x}\,dx$$

$$= 4\sqrt{2}\left[\frac{x^{3/2}}{3/2}\right]_0^2$$

$$= \frac{8\sqrt{2}}{3}[2^{3/2} - 0]$$

$$= \frac{8\sqrt{2}}{3} \times 2\sqrt{2}$$

$$= \frac{32}{3} \text{ sq. units}$$

25.

$$\frac{dy}{dx} = x^3 \operatorname{cosec} y;\ y\ (0) = 0$$

$$\int \frac{dy}{\operatorname{cosec} y} = \int x^3 dx$$

$$\int \sin y\, dy = \int x^3 dx$$

$$-\cos y = \frac{x^4}{4} + C$$

$$-1 = C \qquad (\because\ y = 0, \text{ when } x = 0)$$

$$\cos y = 1 - \frac{x^4}{4}$$

Explanation: Here,

$$\frac{dy}{dx} = x^3 \operatorname{cosec};\ y(0) = 0$$

$$\frac{dy}{\operatorname{cosec} y} = x^3\, dx$$

On integrating both sides, we get

$$\Rightarrow \quad \int \sin y\, dy = \int x^3 dx$$

$$\Rightarrow \quad -\cos y = \frac{x^4}{4} + C$$

Put $x = 0$ and $y = 0$

Then, $-\cos 0° = \dfrac{0^4}{4} + C$

$$\Rightarrow \quad C = -4$$

Then, $-\cos y = \dfrac{x^4}{4} - 1$

$$\Rightarrow \quad \cos y = 1 - \frac{x^y}{4},$$

is the particular solution of given differential equation.

26. Let $\vec{a} = \hat{i} - \hat{j} + \hat{k}$

$$\vec{d} = 4\hat{i} + 5\hat{k}$$

$$\because \quad \vec{a} + \vec{b} = \vec{d} \therefore \vec{b} = \vec{d} - \vec{a}$$

$$= 3\hat{i} + \hat{j} + 4\hat{k}$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & +\hat{k} \\ 1 & -1 & 1 \\ 3 & 1 & 4 \end{vmatrix}$$

$$= -5\hat{i} - 1\hat{j} + 4\hat{k}$$

Area of parallelogram

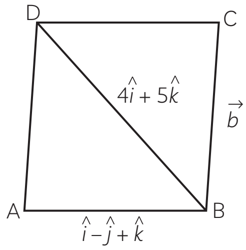
$$= |\vec{a} \times \vec{b}| = \sqrt{25 + 1 + 16}$$

$$= \sqrt{42} \text{ sq. units}$$

Explanation: Here,

$$\vec{a} = \hat{i} - \hat{j} + \hat{k}$$

and $\quad \vec{d} = 4\hat{j} + 5\hat{k}$



$\because \quad \vec{a} + \vec{b} = \vec{d}$

$$\vec{b} = 4\hat{i} + 5\hat{k} - (\hat{i} - \hat{j} + \hat{k})$$

$$= 4\hat{i} - \hat{i} + 5\hat{k} - \hat{k} + \hat{j}$$

$$= 3\hat{i} + \hat{j} + 4\hat{k}$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 3 & 1 & 4 \end{vmatrix}$$

$$= \hat{i}(-4 - 1) - \hat{j}(4 - 3) + \hat{k}(1 + 3)$$

$$= -5\hat{i} - \hat{j} + 4\hat{k}$$

Then, area of parallelogram

$$= |\vec{a} \times \vec{b}|$$

$$= \sqrt{25 + 1 + 16}$$

$$= \sqrt{42} \text{ sq. units}$$

27. Let the normal vector to the plane be $\vec{n}$

Equation of the plane passing through (1, 0, 0), *i.e.*, $\hat{i}$ is

$$(\vec{r} - \hat{i})\cdot\vec{n} = 0 \quad \text{...(1)}$$

$\because$ plane (1) contains the line

$$\vec{r} = \vec{0} + \lambda\hat{j}$$

$\therefore \hat{i}\cdot\vec{n} = 0$ and $\hat{j}\cdot\vec{n} = 0 \Rightarrow \vec{n} = \hat{k}$

Hence equation of the plane is

$$(\vec{r} - \hat{i})\cdot\hat{k} = 0$$

i.e., $\quad \vec{r}\cdot\hat{k} = 0$

Explanation: Let, the normal vector to the plane be $\vec{n}$.

Equation of the plane passing through (1, 0, 0) *i.e.* $\hat{i}$ is

$$(\vec{r} - \hat{i}).\vec{n} = 0 \quad \text{...(1)}$$

$\because$ plane (i) contains the line

$$\vec{r} = \vec{0} + \lambda\hat{j}$$

$\therefore \hat{i}.\vec{n} = 0$ and $\hat{j}.\vec{n} = 0$

$\Rightarrow \vec{n} = k$

Hence, the equation of the plane is

$$(\vec{r} - \hat{i}).\hat{k} = 0$$

i.e., $\quad \vec{r}.\hat{k} = 0$

28. Let x denote the number of milk chocolates drawn

X	P(x)
0	$\frac{4}{6} \times \frac{3}{5} = \frac{12}{30}$
1	$\left(\frac{2}{6} \times \frac{4}{5}\right) \times 2 = \frac{16}{30}$
2	$\frac{2}{6} \times \frac{1}{5} = \frac{2}{30}$

Most likely outcome is getting one chocolate of each type

Explanation: Let x be the number of milk chocolates .Then x = 0,1 or 2.

So, the probability distribution of number of milk chocolates is

x	P(x)
0	$\frac{4}{6} \times \frac{3}{5} = \frac{12}{30}$
1	$\left(\frac{2}{6} \times \frac{4}{5}\right) \times 2 = \frac{16}{30}$
2	$\frac{2}{6} \times \frac{1}{5} = \frac{2}{30}$

Most likely outcome is getting one chocolates of each type.

OR

$$P(\bar{E}|\bar{F}) = \frac{P(\bar{E}\cap\bar{F})}{P(\bar{F})} = \frac{P(\overline{E\cup F})}{P(\bar{F})}$$

$$= \frac{1 - P(E \cup F)}{1 - P(F)}$$

Now, P (E ∪ F) = P (E) + P (F) – P (E ∩ F)

= 0.8 + 0.7 – 0.6 = 0.9

Substituting value of P (E ∪ F) in (1)

$$P(\bar{E}|\bar{F}) = \frac{1 - 0.9}{1 - 0.7} = \frac{0.1}{0.3} = \frac{1}{3}$$

Explanation: Here P(E) = 0.8, P(F) = 0.7, P(E ∩ F) = 0.6

$$P(\bar{E}/\bar{F}) = \frac{P(\bar{E}\cap\bar{F})}{P(\bar{F})} = \frac{1 - P(\overline{E\cup F})}{P(\bar{F})}$$

$$= \frac{1 - P(E\cup F)}{1 - P(F)} \quad ...(1)$$

Now, P(E ∪ F)= P(E) + P(F) – P(E ∩ F)

= 0.8 + 0.7 – 0.6

= 1.5 – 0.6 = 0.9

$$\therefore \quad P(\bar{E}/\bar{F}) = \frac{1 - 0.9}{1 - 0.7} = \frac{0.1}{0.3} = \frac{1}{3}$$

SECTION - IV

29. (i) Reflexive :

Since, $a + a = 2a$ which is even

∴ $(a, a) \in R \ \forall \ a \in Z$

Hence R is reflexive.

(ii) Symmetric :

If $(a, b) \in R$, then $a + b = 2\lambda$

$\Rightarrow b + a = 2\lambda$

$\Rightarrow$ $(b, a) \in R$, Hence R is symmetric

(iii) Transitive :

If $(a, b) \in R$ and $(b, c) \in R$

then $a + b = 2\lambda$ ...(1)

and $b + c = 2\mu$ ...(2)

Adding (1) and (2) we get

$a + 2b + c = 2(\lambda + \mu)$

$\Rightarrow$ $a + c = 2(\lambda + \mu - b)$

$\Rightarrow$ $a + c = 2k$

where $\lambda + \mu - b = k \Rightarrow (a, c) \in R$

Hence R is transitive

[0] = {.... – 4, – 2, 0, 2, 4 ....}

Explanation:

(i) Reflexive :

Since, $a + a = 2a$ which is even

∴ $(a, a) \in R \ \forall \ a \in Z$

Hence, R is reflexive.

(ii) Symmetric :

If $(a, b) \in R$, then $a + b = 2\lambda \Rightarrow b + a = 2\lambda$

$\Rightarrow$ $(b, a) \in R$,

Hence, R is symmetric

(iii) Transitive :

If $(a, b) \in R$ and $(b, c) \in R$

Then $a + b = 2\lambda$ ...(1)

and $b + c = 2\mu$ ...(2)

On addding (1) and (2) we get

$a + 2b + c = 2(\lambda + \mu)$

$\Rightarrow$ $a + c = 2(\lambda + \mu - b)$

$\Rightarrow$ $a + c = 2k$

where $\lambda + \mu - b = k \Rightarrow (a, c) \in R$

Hence, R is transitive.

Equivalence class containting

[0] = {.... – 4, – 2, 0, 2, 4 ....}

30. Let $u = e^{x\sin^2 x}$ and $v = (\sin x)^x$

so that $y = u + v$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \quad ...(1)$$

Now, $u = e^{x\sin^2 x}$,

Differentiating both sides w.r.t. x, we get

$$\Rightarrow \quad \frac{du}{dx} = e^{x\sin^2 x}[x(\sin 2x) + \sin^2 x] \quad ...(2)$$

Also, $v = (\sin x)^x$

$\Rightarrow$ $\log v = x \log(\sin x)$

Differentiating both sides w.r.t. x, we get

$$\frac{1}{v}\frac{dv}{dx} = x\cot x + \log(\sin x)$$

$$\frac{dv}{dx} = (\sin x)^x[x\cot x + \log(\sin x)] \quad ...(3)$$

Substituting from (2), sin $2x$ (3) in (1) we get

$$\frac{dy}{dx} = e^{x\sin^2 x}[x\sin 2x + \sin^2 x] + (\sin x)^x[x\cot x + \log(\sin x)]$$

Explanation: Here,

$$y = e^{x\sin^2 x} + (\sin x)^x$$

Let $\quad u = e^{x \sin^2 x}, v = (\sin x)^x$

So, $\quad y = u + v$

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \quad ...(1)$$

Now, $\quad u = e^{x \sin^2 x}$

On differentiating both sides we get

$$\frac{du}{dx} = e^{x \sin^2 x} [x (2 \sin x \cos x) + \sin^2 x]$$

$$= e^{x \sin^2 x} [x \sin^2 x + \sin 2x] \quad ...(2)$$

Also, $\quad v = (\sin x)^x$

$$\log v = x \log (\sin x)$$

[Taking log on both sides]

Differentiating with sides w.r.t. x, we get

$$\frac{1}{v} \times \frac{dv}{dx} = \frac{x \times \cos x}{\sin x} + \log(\sin x)$$

$$\Rightarrow \frac{dv}{dx} = (\sin x)^x [x \cot x + \log (\sin x)] \quad ...(3)$$

Substituting values of (2) and (3) in equation (1), we get

$$\frac{dy}{dx} = e^{x \sin^2 x} [x \sin 2x + \sin^2 x]$$
$$+ (\sin x)^x [x \cot x + \log (\sin x)]$$

31.
$$\text{RHD} = \underset{h \to 0}{Lt} \frac{f(1+h) - f(1)}{h}$$
$$= \underset{h \to 0}{Lt} \frac{[1+h] - [1]}{h}$$
$$= \underset{h \to 0}{Lt} \frac{(1-1)}{h} = 0$$
$$\text{LHD} = \underset{h \to 0}{Lt} \frac{f(1-h) - f(1)}{-h}$$
$$= \underset{h \to 0}{Lt} \frac{[1-h] - [1]}{-h} = \underset{h \to 0}{Lt} \frac{0-1}{-h}$$
$$= \underset{h \to 0}{Lt} \frac{1}{h} = \infty$$

Since, RHD ≠ LHD

Therefore $f(x)$ is not differentiable at $x = 1$

Explanation: $\text{RHD} = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$

$$= \lim_{h \to 0} \frac{[1+h] - [1]}{h}$$

$$= \lim_{h \to 0} \frac{1-1}{h} = 0$$

$$\text{LHD} = \lim_{h \to 0} \frac{f(1-h) - f(1)}{-h}$$

$$= \lim_{h \to 0} \frac{[1-h] - [1]}{-h}$$

$$= \lim_{h \to 0} \frac{0-1}{-h}$$

$$= \lim_{h \to 0} \frac{1}{h} = \infty$$

Since, $\quad$ RHD ≡ ≠ LHD

Therefore, $f(x)$ is not differentiable at $x = 1$.

OR

$$y = b \tan \theta$$

$$\Rightarrow \frac{dy}{d\theta} = b \sec^2 \theta \quad ...(1)$$

$$x = a \sec \theta$$

$$\Rightarrow \frac{dx}{d\theta} = a \sec \theta \tan \theta \quad ...(2)$$

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{b \sec^2 \theta}{a \sec\theta \tan\theta} = \frac{b}{a} \operatorname{cosec} \theta$$

Differentiating both sides w.r.t x, we get

$$\frac{d^2y}{dx^2} = \frac{-b}{a} \operatorname{cosec} \theta \cot\theta \times \frac{d\theta}{dx}$$

$$= \frac{-b}{a} \operatorname{cosec} \theta \cot\theta \times \frac{1}{a \sec\theta \tan\theta}$$

$$= \frac{-b}{a.a} \cot^3 \theta$$

$$\left.\frac{d^2y}{dx^2}\right]_{\theta = \frac{\pi}{6}} = \frac{-b}{a.a}\left[\cot \frac{\pi}{6}\right]^3 = \frac{-b}{a.a}(\sqrt{3})^3$$

$$= -\frac{3\sqrt{3}b}{a.a}$$

Explanation:

Here, $\quad x = a \sec \theta$

$$\Rightarrow \frac{dx}{d\theta} = a \sec \theta \tan \theta \quad ...(1)$$

Also, $\quad y = b \tan \theta$

$$\Rightarrow \frac{dy}{d\theta} = b \sec^2 \theta \qquad ...(2)$$

Then, $$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{b \sec^2\theta}{a \sec\theta \tan\theta}$$

$$= \frac{b}{a}\frac{\sec\theta}{\tan\theta}$$

$$= \frac{b}{a} \times \frac{1}{\cos\theta} \times \frac{\cos\theta}{\sin\theta}$$

$$= \frac{b}{a} \times \operatorname{cosec}\theta$$

Differentiating both sides w.r.t x, we get

$$\frac{d^2y}{dx^2} = \frac{-b}{a}\operatorname{cosec}\theta \cot\theta \times \frac{d\theta}{dx}$$

$$= \frac{-b}{a}\operatorname{cosec}\theta \cot\theta \times \frac{1}{a \sec\theta \tan\theta}$$

$$= \frac{-b}{a.a} \times \frac{1}{\sin\theta} \times \frac{\cos\theta}{\sin\theta} \times \frac{\cos\theta}{\sin\theta} \times \cos\theta$$

$$= \frac{-b}{a.a} \times \frac{\cos^3\theta}{\sin^3\theta}$$

$$= \frac{-b}{a^2}\cot^3\theta$$

$$\left.\frac{d^2y}{dx_2}\right]_{\theta = \frac{\pi}{6}} = \frac{-b}{a^2}\left[\cot\frac{\pi}{6}\right]^3$$

$$= \frac{-b}{a^2}\left[\sqrt{3}\right]^3 = \frac{-3\sqrt{3}b}{a^2}$$

32. $f(x) = \tan x - 4x$

$f'(x) = \sec^2 x - 4$

(a) For $f(x)$ to be strictly increasing

$f'(x) > 0$

$\Rightarrow \quad \sec^2 x - 4 > 0$

$\Rightarrow \quad \sec^2 x > 4$

$\Rightarrow \quad \cos^2 x < \frac{1}{4} \Rightarrow \cos^2 x < \left(\frac{1}{2}\right)^2$

$\Rightarrow \quad -\frac{1}{2} < \cos x < \frac{1}{2} \Rightarrow \frac{\pi}{3} < x < \frac{\pi}{2}$

(b) For $f(x)$ to be strictly decreasing

$f'(x) < 0$

$\Rightarrow \quad \sec^2 x - 4 > 0$

$\Rightarrow \quad \sec^2 x < 4$

$\Rightarrow \quad \cos^2 x > \frac{1}{4}$

$\Rightarrow \quad \cos^2 x > \left(\frac{1}{2}\right)^2$

$\Rightarrow \quad \cos x > \frac{1}{2} \qquad \left[\because x \in \left(0, \frac{\pi}{2}\right)\right]$

$\Rightarrow \quad 0 < x < \frac{\neq}{3}$

Explanation: $f(x) = \tan x - 4x$

$f'(x) = \sec^2 x - 4$

(a) For $f(x)$ to be strictly increasing

$f'(x) > 0$

$\Rightarrow \quad \sec^2 x - 4 > 0$

$\Rightarrow \quad \sec^2 x < 4$

$\Rightarrow \quad \cos^2 x < \frac{1}{4}$

$\Rightarrow \quad \cos^2 x < \left(\frac{1}{2}\right)^2$

$\Rightarrow \quad \frac{-1}{2} < \cos x < \frac{1}{2}$

$\Rightarrow \quad -\frac{\neq}{3} < x < \frac{\neq}{3}$

But $\quad x \in \left[0, \frac{\pi}{2}\right]$

$\therefore \quad 0 < x < \frac{\pi}{3}$

(b) For $f(x)$ to be strictly decreasing

$f'(x) < 0$

$\Rightarrow \quad \sec^2 x - 4 < 0$

$\Rightarrow \quad \sec^2 x < 4$

$\Rightarrow \quad \cos^2 x > \frac{1}{4}$

$\Rightarrow \quad \cos^2 x > \left(\frac{1}{2}\right)^2$

$\Rightarrow \quad \cos x > \frac{1}{2} \qquad \left[\because x \in \left(0, \frac{\pi}{2}\right)\right]$

$\Rightarrow \quad 0 < x < \frac{\neq}{3}$

$\Rightarrow \quad x > \frac{\neq}{3}$

$\because \quad x \in \left[0, \frac{\pi}{2}\right]$

$\therefore \quad \frac{\neq}{3} < x < \frac{\pi}{2}$

33. Put $x^2 = y$ to make partial fractions

$$\frac{x^2+1}{(x^2+2)(x^2+3)} = \frac{y+1}{(y+2)(y+3)}$$

$$= \frac{A}{y+2} + \frac{B}{y+3}$$

$$y + 1 = A(y+3) + B(y+2) \qquad ...(1)$$

Comparing coeffcients of y and constant terms on both sides of (1) we get

A + B = 1 and 3A + 2B = 1

Solving, we get A = – 1, B = 2

$$\int \frac{x^2+1}{(x^2+2)(x^2+3)} dx$$

$$= \int \frac{-1}{x^2+2} dx + 2\int \frac{1}{x^2+3} dx$$

$$= -\frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

$$+ \frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{x}{\sqrt{3}}\right) + C$$

Explanation: Let $I = \int \frac{x^2+1}{(x^2+2)(x^2+3)} dx$.

Put $x^2 = y$ to make partial functions

i.e. $\frac{x^2+1}{(x^2+2)(x^2+3)} =$

$$\frac{y+1}{(y+2)(y+3)} = \frac{A}{(y+2)} + \frac{B}{(y+3)}$$

$$y + 1 = A(y+3) + B(y+2) \qquad ...(1)$$

On comparing coeffcient of 'y' and constant terms on both sides of (1), we get

A + B = 1 and 3A + 2B = 1

On solving we get, A = – 1, B = 2

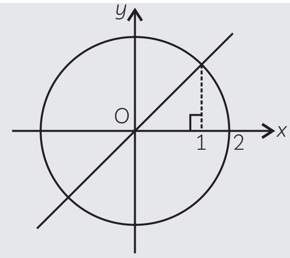
$$\int \frac{x^2+1}{(x^2+2)(x^2+3)} dx = \int \frac{-1}{x^2+2} dx + 2\int \frac{1}{x^2+3} dx$$

$$= \frac{-1}{\sqrt{2}} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right) + \frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{x}{\sqrt{3}}\right) + C$$

34. Solving $y = \sqrt{3}x$ and $x^2 + y^2 = 4$

We get $x^2 + 3x^2 = 4$

$x^2 = 1 \Rightarrow x = 1$



Required Area

$$= \sqrt{3}\int_0^1 x\,dx + \int_1^2 \sqrt{2^2 - x^2}\,dx$$

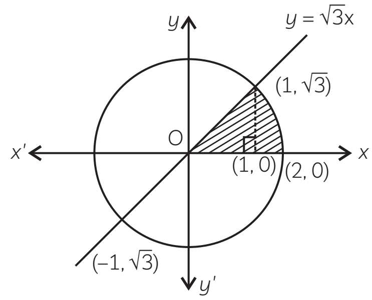
$$= \frac{\sqrt{3}}{2}[x^2]_0^1 + \left[\frac{x}{2}\sqrt{2^2 - x^2} + 2\sin^{-1}\left(\frac{x}{2}\right)\right]_1^2$$

$$= \frac{\sqrt{3}}{2} + \left[2 \times \frac{\pi}{2} - \frac{\sqrt{3}}{2} - 2 \times \frac{\pi}{6}\right]$$

$$= \frac{2\neq}{3} \text{ sq. units}$$

Explanation: Given, curves are

$$x^2 + y^2 = 4,\ y = \sqrt{3}\,x$$



On solving , we get

$$x^2 + (\sqrt{3}x)^2 = 4$$

$$\Rightarrow \qquad 4x^2 = 4$$

$$\Rightarrow \qquad x^2 = 1 \Rightarrow x = \pm 1$$

Then, Required Area

$$= \int_0^1 y_{line}\,dx + \int_1^2 y_{circle}\,dx$$

$$= \int_0^1 \sqrt{3}x\,dx + \int_1^2 \sqrt{2^2 - x^2}\,dx$$

$$= \frac{\sqrt{3}}{2}[x^2]_0^1 + \left[\frac{x}{2}\sqrt{4-x^2} + 2\sin^{-1}\left(\frac{x}{2}\right)\right]_1^2$$

$$= \frac{\sqrt{3}}{2} + \left[2 \times \frac{\pi}{2} - \frac{\sqrt{3}}{2} - 2 \times \frac{\pi}{6}\right]$$

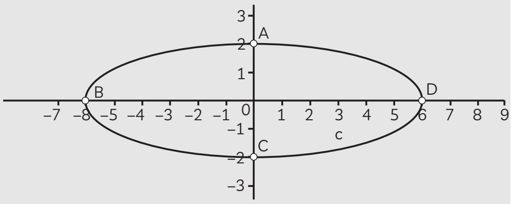
$$= \frac{\sqrt{3}}{2} + \left[\pi - \frac{\sqrt{3}}{2} - \frac{\pi}{3}\right]$$

$$= \pi - \frac{\pi}{3} = \frac{2\pi}{3}$$

Hence, the area of region bounded by curves is $\frac{2\neq}{3}$ sq. units.

OR

Required Area = $\frac{4}{3}\int_0^6 \sqrt{6^2 - x^2}dx$



$$= \frac{4}{3}\left[\frac{x}{2}\sqrt{6^2 - x^2} + 18\sin^{-1}\left(\frac{x}{6}\right)\right]_0^6$$

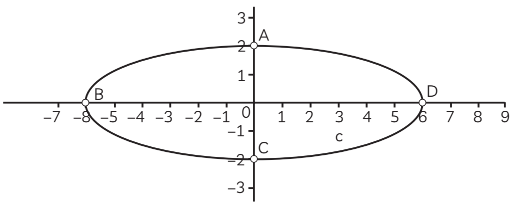
$$= \frac{4}{3}\left[18 \times \frac{\pi}{2} - 0\right] = 12\,\pi \text{ sq. units}$$

Explanation: Given, equation of ellipse, is

$x^2 + 9y^2 = 36$

$$\Rightarrow \quad = \frac{x^2}{36} + \frac{y^2}{4} = 1$$

$$\Rightarrow \quad = \frac{x^2}{6^2} + \frac{y^2}{2^2} = 1$$



Then, required area,

$$A = 4\equiv_{0}^{6} y.dx$$

$$= \frac{4}{3}\int_0^6 \sqrt{6^2 - x^2}dx$$

$$\left[\because y^2 = \frac{36 - x^2}{9} \text{ or } y = \frac{\sqrt{36 - x^2}}{3}\right]$$

$$= \frac{4}{3}\left[\frac{x}{2}\sqrt{6^2 - x^2} + 18\sin^{-1}\left(\frac{x}{6}\right)\right]_0^6$$

$$= \frac{4}{3}\left[18 \times \frac{\pi}{2} - 0\right]$$

$$= \frac{4}{3} \times 9\neq$$

$$= 12\,\pi \text{ sq. units}$$

Hence, the required area is $12\,\pi$ sq. units.

35. The given differential equation can be written as

$$\frac{dy}{dx} = \frac{y + 2x^2}{x}$$

$$\Rightarrow \quad \frac{dy}{dx} - \frac{1}{x}y = 2x$$

Here, $\quad P = -\frac{1}{x}, \; Q = 2x$

$$\text{IF} = e^{\int P dx} = e^{-\int \frac{1}{x}dx}$$

$$= e^{-\log x} = \frac{1}{x}$$

The solution is :

$$y \times \frac{1}{x} = \int\left(2x \times \frac{1}{x}\right)dx$$

$$\Rightarrow \quad \frac{y}{x} = 2x + c$$

$$\Rightarrow \quad y = 2x^2 + cx$$

Explanation: Here,

$x\,dx - (y + 2x^2)\,dx = 0$

Then $\quad x\,dy = (y + 2x^2)\,dx$

$$\frac{dy}{dx} = \frac{y + 2x^2}{x}$$

$$\Rightarrow \quad \frac{dy}{dx} - \frac{1y}{x} = 2x$$

Here, $\quad P = \frac{-1}{x} \; Q = 2x$

$$\text{I.F.} = e^{\int P.dx} = e^{\int \frac{-1}{x}dx} = e^{-\log x} = \frac{1}{x}$$

Then, its solution is

$$y \times \frac{1}{x} = \int 2x \times \frac{1}{x}dx + c$$

$$\frac{y}{x} = 2x + c$$

$$y = 2x^2 + cx,$$

SECTION - V

36. $|A| = 1(-1-2) - 2(-2-0) = -3+4 = 1$

A is non-singular, therefore A^{-1} exists

$$\text{Adj } A = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$$

$$A^{-1} = \frac{\text{Adj } A}{|A|}(\text{Adj } A)$$

$$= \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$$

The given equations can be written as :

$$\begin{bmatrix} 1 & -2 & 0 \\ 2 & -1 & -1 \\ 0 & -2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$$

Which is of the form A'X = B

$\Rightarrow$ $X = (A')^{-1} B = (A^{-1})' B$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3 & 2 & 2 \\ -2 & 1 & 1 \\ -4 & 2 & 3 \end{bmatrix}\begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix} = \begin{bmatrix} 0 \\ -5 \\ -3 \end{bmatrix}$$

$\Rightarrow$ $x = 0, \ y = -5, z = -3$

Explanation: Here, $A = \begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$

Now, $|A| = (-1-2) - 2(-2-0) + 0(2+0)$

$= 1(-3) + 4 + 0$

$= 1$

Then, A is non-singular, therefore A^{-1} exist.

Now, $a_{11} = (-1)^{1+1}(-1-2) = -3$

$a_{12} = (-1)^{1+2}(-2-0) = 2$

$a_{13} = (-1)^{1+3}(2-0) = 2$

$a_{21} = (-1)^{2+1}(2-0) = -2$

$a_{22} = (-1)^{2+2}(1-0) = 1$

$a_{23} = (-1)^{2+3}(-1-0) = 1$

$a_{31} = (-1)^{3+1}(-4-0) = -4$

$a_{32} = (-1)^{3+2}(-2-0) = 2$

$a_{33} = (-1)^{3+3}(-1+4) = 3$

$$\text{Adj } A = \begin{bmatrix} -3 & 2 & 2 \\ -2 & 1 & 1 \\ -4 & 2 & 3 \end{bmatrix}^T$$

$$= \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$$

Then, $A^{-1} = \frac{1}{|A|} \text{adj } A = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$

The given equation can be written as

$$\begin{bmatrix} 1 & -2 & 0 \\ 2 & -1 & -1 \\ 0 & -2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$$

which is of the form

$A'X = B$

$\Rightarrow$ $X = (A')^{-1} B = (A^{-1})' B$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}^T \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & 2 & 2 \\ -2 & 1 & 1 \\ -4 & 2 & 3 \end{bmatrix}\begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$$

$$= \begin{bmatrix} -30+16+14 \\ -20+8+7 \\ -40+16+21 \end{bmatrix} = \begin{bmatrix} 0 \\ -5 \\ -3 \end{bmatrix}$$

$\therefore$ $x = 0, y = -5, z = -3$

OR

$$AB = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

$AB = 6I$

$$A\left(\frac{1}{6}B\right) = I \Rightarrow A^{-1} = \frac{1}{6}(B)$$

The given equations can be written as

$$\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$$

$AX = D$, where $D = \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$

$$\Rightarrow \quad X = A^{-1} D$$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}\begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$$

$$= \frac{1}{6}\begin{bmatrix} 12 \\ -6 \\ 24 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$$

$$\Rightarrow \quad x = 2,\ y = -1,\ z = 4$$

Explanation: Here,

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$$

and $B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$

Then

$$AB = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}_{3\times 3}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}_{3\times 3}$$

$$= \begin{bmatrix} 2+4+0 & 2-2-0 & -4+4+0 \\ 4-12+8 & 4+6-4 & -8-12+20 \\ 0-4+4 & 0+2-2 & 0-4+10 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

$$AB = 6I$$

$$A\left(\frac{1}{6}B\right) = I$$

$$A^{-1} = \frac{1}{6}B$$

The given, equations can be written in matrix form as,

$$\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$$

$AX = D$ where, $D = \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$

$$X = A^{-1} D$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}\begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$$

$$= \frac{1}{6}\begin{bmatrix} 12 \\ -6 \\ 24 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$$

$\therefore\ x = 2, y = -1, z = 4$

37. We have $a_1 = 3\hat{i} + 2\hat{j} - 4\hat{k}$

$$b_1 = \hat{i} + 2\hat{j} + 2\hat{k}$$

$$a_2 = 5\hat{i} - 2\hat{j}$$

$$b_2 = 3\hat{i} + 2\hat{j} + 6\hat{k}$$

$$\vec{a_2} - \vec{a_1} = 2\hat{i} - 4\hat{j} + 4\hat{k}$$

$$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 2 \\ 3 & 2 & 6 \end{vmatrix}$$

$$= \hat{i}(12-4) - \hat{j}(6-6) + \hat{k}(2-6)$$

$$\vec{b_1} \times \vec{b_2} = 8\hat{i} + 0\hat{j} - 4\hat{k} = 8\hat{i} - 4\hat{k}$$

$$\because (\vec{b_1} \times \vec{b_2}).(\vec{a_2} \times \vec{a_1}) = 16 - 16 = 0$$

$\therefore$ The lines are intersecting and the shortest distance between the lines is 0.

Now for point of intersection

$$3\hat{i} + 2\hat{j} - 4\hat{k} + \lambda(\hat{i} + 2\hat{j} + 2\hat{k})$$
$$= 5\hat{i} - 2\hat{j} + \mu(3\hat{i} + 2\hat{j} + 6\hat{k})$$

$$3 + \lambda = 5 + 3\mu \quad \text{...(1)}$$

$$2 + 2\lambda = -2 + 2\mu \quad \text{...(2)}$$

$$-4 + 2\lambda = 6\mu \quad \text{...(3)}$$

Solving (1) and (2) we get, $\mu = -2$ and $\lambda = -4$

Substituting in equation of line we get

$$\vec{r} = 5\hat{i} - 2\hat{j} + (-2)(3\hat{i} + 2\hat{j} + 6\hat{k})$$

$$= -\hat{i} - 6\hat{j} - 12\hat{k}$$

Point of intersection is (–1, – 6, – 12)

Explanation: Here, given lines are

$$\vec{r} = (3\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(\hat{i} + 2\hat{j} + 2\hat{k})$$

and $\vec{r} = 5\hat{i} - 2\hat{j} + \mu(3\hat{i} + 2\hat{j} + 6\hat{k})$

Then, we have

$$a_1 = 3\hat{i} + 2\hat{j} - 4\hat{k}$$

$$b_1 = \hat{i} + 2\hat{j} + 2\hat{k}$$

and $a_2 = 5\hat{i} - 2\hat{j}$

$$b_2 = 3\hat{i} + 2\hat{j} + 6\hat{k}$$

$$\vec{a_2} - \vec{a_1} = (5\hat{i} - 2\hat{j}) - (3\hat{i} + 2\hat{j} - 4\hat{k})$$

$$= 2\hat{i} - 4\hat{j} + 4\hat{k}$$

$$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 2 \\ 3 & 2 & 6 \end{vmatrix}$$

$$= \hat{i}(12 - 4) - \hat{j}(6 - 6) + \hat{k}(2 - 6)$$

$$= 8\hat{i} - 4\hat{k}$$

$$\because (\vec{b_1} \times \vec{b_2}).(\vec{a_2} - \vec{a_1})$$

$$= (8\hat{i} - 4\hat{k}).(2\hat{i} - 4\hat{j} + 4\hat{k})$$

$$= 16 - 16 = 0$$

$\therefore$ The lines are intersecting and the shortest distance between the lines is 0.

Now, for point of intersection

$$3\hat{i} + 2\hat{j} - 4\hat{k} + \lambda(\hat{i} + 2\hat{j} + 2\hat{k})$$

$$= 5\hat{i} - 2\hat{j} + \mu(3\hat{i} + 2\hat{j} + 6\hat{k})$$

$(3 + \lambda) = 5 + 3\mu$ ...(1)

$2 + 2\lambda = -2 + 2\mu$ ...(2)

$-4 + 2\lambda = 6u$ ...(3)

On solving (1) and (2), we get

$\mu = -2$ and $\lambda = -4$

Substituting it in equation of line, we get

$$\vec{r} = 5\hat{i} - 2\hat{j} + (-2)(3\hat{i} + 2\hat{j} + 6\hat{k})$$

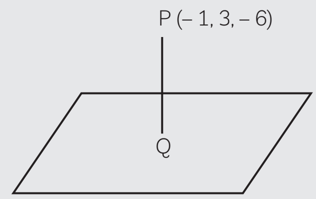
$$= -\hat{i} - 6\hat{j} - 12\hat{k}$$

Then, point of intersection is (– 1, – 6, – 12).

OR

Let P be the given point and Q be the foot of the perpendicular.

Equation of PQ $\dfrac{x+1}{2} = \dfrac{y-3}{1} = \dfrac{z+6}{-2}$



Let coordinates of Q be $(2\lambda - 1, \lambda + 3, -2\lambda - 6)$

Since Q lies in the plane

$2x + y - 2z + 5 = 0$

$\therefore 2(2\lambda - 1) + (\lambda + 3) - 2(-2\lambda - 6) + 5 = 0$

$4\lambda - 2 + \lambda + 3 + 4\lambda + 12 + 5 = 0$

$9\lambda + 18 = 0 \Rightarrow \lambda = -2$

$\therefore$ coordinates of Q are (– 5, 1, – 2)

Length of the perpendicular

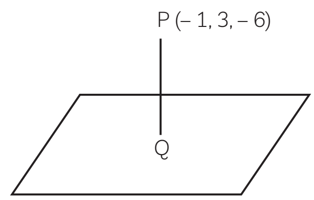
$$= \sqrt{(-5+1)^2 + (1-3)^2 + (-2+6)^2}$$

$$= 6 \text{ units}$$

Explanation: Let, P be the given point with coordinates (– 1, 3, – 6) and Q be the foot of perpendicular.

Equation of PQ

$$\frac{x+1}{2} = \frac{y-3}{1} = \frac{z+6}{-2} = \lambda$$



Let, the coordinates of

Q $(2\lambda - 1, \lambda + 3, -2\lambda - 6)$

Since, point Q lies on the plane

$$2x + y - 2z + 5 = 0$$

$\therefore\ 2(2\lambda + 1) + (\lambda + 3) - 2(-2\lambda - 6) + 5 = 0$

$\Rightarrow\ 4\lambda - 2 + \lambda + 3 + 4\lambda + 12 + 5 = 0$

$\Rightarrow\ 9\lambda + 18 = 0$

$\Rightarrow\ \lambda = -2$

$\therefore$ coordinates of Q are

$$(2 \times -2) - 1, -2 + 3, -2 \times -2) - 6)$$

$$= (-5, 1, -2)$$

$\therefore$ Length of perpendicular

$$= \sqrt{(-5+1)^2 + (1-3)^2 + (-2+6)^2}$$

$$= \sqrt{(-4)^2 + (-2)^2 + (4)^2}$$

$$= \sqrt{16 + 4 + 16} = \sqrt{36}$$

$$= 6 \text{ units}$$

38. Maximize $Z = 3x + y$

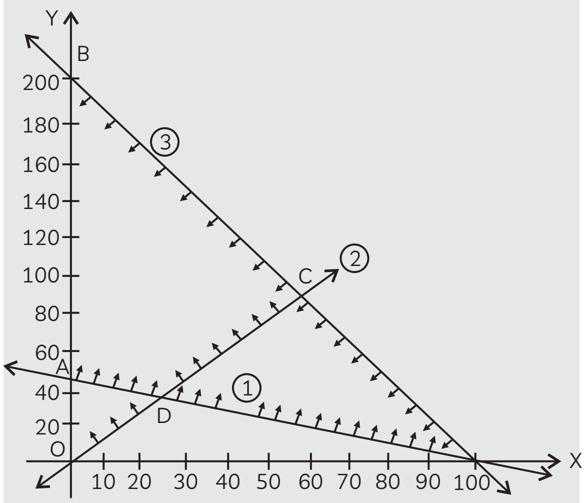
Subject to constraints

$x + 2y \geq 100$ ...(1)

$2x - y \leq 0$ ...(2)

$2x + y \leq 200$ ...(3)

$x, y \geq 0$



Corner Points	Z = 3x + y
A (0, 50)	50
B (0, 200)	200
C (50, 100)	250
D (20, 40)	100

max $z = 250$ at $x = 50$, $y = 100$

Explanation: Max $Z = 3x + y$

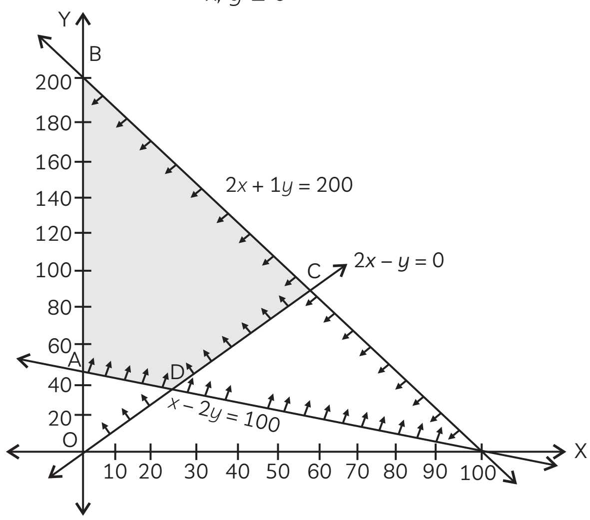
Subject to constraints

$x + 2y \geq 100$ ...(1)

$2x - y \leq 0$ ...(2)

$2x + y \leq 200$ ...(3)

$x, y \geq 0$



Change in equations into equations. For equation (1)

$x + 2y = 100$

x	0	100
y	50	0

For equation (2)

$2x - y = 0$

x	0	30
y	0	60

For equation of (3)

$2x + y = 200$

x	100	0
y	0	200

Corner Points	*Objective function Max Z = x + 2y*
A(0, 50)	100
B(0, 200)	400 (Maximum)
C(50, 100)	250 (Maximum)
D(20, 40)	100

$\therefore$ Z is maximum at x = 0, y = 400

OR

Corner Points	*Z = 3x – 4y*
O (0, 0)	0
A (0, 8)	– 32 (Minimum)
B (4, 10)	– 28
C (6, 8)	– 14
D (6, 5)	– 2
E (4, 0)	12 (Maximum)

Max. Z = 12 at E (4, 0)

Min. Z = – 32 at A (0, 8)

(ii) Since maximum value of Z occurs at B(4, 10) and C(6, 8)

$\therefore \quad 4p + 10q = 6p + 8q$

$\Rightarrow \quad 2q = 2p$

$\Rightarrow \quad p = q$

Number of optimal solution are infinite.

Explanation: (i)

Corner Points	*Z = 3x – 4y*
O (0, 0)	0
A (0, 8)	– 32 (Maximum)
B (4, 10)	– 28
C (6, 8)	– 14
D (6, 5)	– 2
E (4, 0)	12 (Maximum)

Max Z = 12 at E (4, 0)

Min Z = – 32 at A (0, 8)

(ii) Since, maximum value of Z occurs at B(4, 10) and C(6, 8)

and objective function, Z = $px + qy$

then, $4p + 10q = 6p + 8q$

$\Rightarrow \quad 2q = 2p$

$\Rightarrow \quad p = q$

Number of optimal solution are infinite.

2

CBSE

2020 (65/2/1)

(REDUCED SYLLABUS)

MATHEMATICS

Time Allowed: 3 Hours *Maximum Marks: 80*

General Instructions:

Read the following instructions very carefully and strictly follow them:

(i) This question paper comprises **four** Sections A, B, C and D. This question paper carries **36** questions. **All** questions are compulsory.

(ii) **Section A** - Questions no. **1** to **20** comprises of **20** questions of **1** mark each.

(iii) **Section B** - Questions no. **21** to **26** comprises of **6** questions of **2** marks each.

(iv) **Section C** - Questions no. **27** to **32** comprises of **6** questions of **4** marks each.

(v) **Section D** - Questions no. **33** to **36** comprises of **4** questions of **6** marks each.

(vi) There is no overall choice in the question paper. However, an internal choice has been provided in 3 questions of one mark, 2 questions of two marks, 2 questions of four marks and 2 questions of six marks. Only one of the choices in such questions have to be attempted.

(vii) In addition to this separate instructions are given with each section and question, wherever necessary.

(viii) Use of calculators is **not** permitted.

SECTION - A **20 Marks**

Directions Q. (1-10): Select the most appropriate option from those given below each question:

1. The relation R in the set {1, 2, 3} given by R = {(1, 2), (2, 1), (1, 1)} is:

(a) symmetric and transitive, but not reflexive

(b) reflexive and symmetric, but not transitive

(c) symmetric, but neither reflexive nor transitive

(d) an equivalence relation 1

***2.** $\tan^{-1} 3 + \tan^{-1} \lambda = \tan^{-1}\left(\frac{3+\lambda}{1-3\lambda}\right)$ is valid for what values of λ ?

(a) $\lambda \in \left(-\frac{1}{3}, \frac{1}{3}\right)$ (b) $\lambda > \frac{1}{3}$

(c) $\lambda < \frac{1}{3}$ (d) All real values of λ

*Not examinable for 2021 exam.

3. If A is a non-singular square matrix of order 3 such that $A^2 = 3A$, then value of $|A|$ is:

(a) -3 (b) 3

(c) 9 (d) 27

4. The function $f: R \to R$ given by $f(x) = -|x-1|$ is :

(a) continuous as well as differentiable at $x = 1$

(b) not continuous but differentiable at $x = 1$

(c) continuous but not differentiable at $x = 1$

(d) neither continuous nor differentiable at $x = 1$ 1

5. Let A = {1, 3, 5}. Then the number of equivalence relations in A containing (1, 3) is:

(a) 1 (b) 2

(c) 3 (d) 4 1

6. The interval in which the function f given by $f(x) = x^2 e^{-x}$ is strictly increasing, is:

(a) $(-\infty, \infty)$ (b) $(-\infty, 0)$

(c) $(2, \infty)$ (d) $(0, 2)$ 1

7. If $|\vec{a}| = 4$ and $-3 \le \lambda \le 2$, then $|\lambda \vec{a}|$ lies in:

(a) [0, 12] (b) [2, 3]

(c) [8, 12] (d) [– 12, 8] 1

***8.** The vectors $3\hat{i} - \hat{j} + 2\hat{k}$, $2\hat{i} + \hat{j} + 3\hat{k}$ and $\hat{i} + \lambda\hat{j} - \hat{k}$ are coplanar if value of λ is:

(a) -2 (b) 0

(c) 2 (d) Any real number 1

9. The area of a triangle formed by vertices O, A and B, where $\vec{OA} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{OB} = -3\hat{i} - 2\hat{j} + \hat{k}$ is:

(a) $3\sqrt{5}$ sq. units (b) $5\sqrt{5}$ sq. units

(c) $6\sqrt{5}$ sq. units (d) 4 sq. units 1

10. The coordinates of the foot of the perpendicular drawn from the point (2, – 3, 4) on the y-axis is:

(a) (2, 3, 4) (b) (– 2, – 3, – 4)

(c) (0, – 3, 0) (d) (2, 0, 4) 1

Directions Q. (11-15): Fill in the blanks

11. The range of the principal value branch of the function $y = \sec^{-1} x$ is 1

OR

The principal value of $\cos^{-1}\left(-\frac{1}{2}\right)$ is 1

12. Given a skew-symmetric matrix $A = \begin{bmatrix} 0 & a & 1 \\ -1 & b & 1 \\ -1 & c & 0 \end{bmatrix}$, the value of $(a + b + c)^2$ is 1

13. The distance between parallel planes $2x + y - 2z - 6 = 0$ and $4x + 2y - 4z = 0$ is .... units.

OR

If $P(1, 0, -3)$ is the foot of the perpendicular from the origin to the plane, then the cartesian equation of the plane is 1

***14.** If the radius of the circle is increasing at the rate of 0.5 cm/s, then the rate of increase of its circumference is 1

***Not examinable for 2021 exam.**

15. The corner points of the feasible region of an LPP are (0, 0), (0, 8), (2, 7), (5, 4) and (6, 0). The maximum profit $P = 3x + 2y$ occurs at the point 1

Directions Q. (16-20): Very Short Answer Type Questions

16. Differentiate $\sec^2(x^2)$ with respect to x^2. 1

OR

If $y = f(x^2)$ and $f'(x) = e^{\sqrt{x}}$, then find $\frac{dy}{dx}$. 1

17. Find the value of k, so that the function $f(x) = \begin{cases} kx^2 + 5 & \text{if } x \le 1 \\ 2 & \text{if } x > 1 \end{cases}$ is continuous at $x = 1$. 1

18. Evaluate : $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x \cos^2 x \, dx$ 1

19. Find the general solution of the differential equation $e^{y-x} \frac{dy}{dx} = 1$. 1

20. Find the coordinates of the point where the line $\frac{x-1}{3} = \frac{y+4}{7} = \frac{z+4}{2}$ cuts the xy-plane. 1

SECTION - B **12 Marks**

21. If $A = \begin{bmatrix} -3 & 2 \\ 1 & -1 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, find scalar k so that $A^2 + I = kA$. 2

22. If $f(x) = \sqrt{\frac{\sec x - 1}{\sec x + 1}}$, find $f'\left(\frac{\pi}{3}\right)$. 2

OR

Find $f'(x)$ if $f(x) = (\tan x)^{\tan x}$. 2

23. Find : $\equiv \frac{\tan^3 x}{\cos^3 x} dx$. 2

24. Find a vector $\vec{r}$ equally inclined to the three axes and whose magnitude is $3\sqrt{3}$ units. 2

OR

Find the angle between unit vectors $\vec{a}$ and $\vec{b}$ so that $\sqrt{3}\,\vec{a} - \vec{b}$ is also a unit vector. 2

25. Find the points of intersection of the line $\vec{r} = 2\hat{i} - \hat{j} + 2\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$ and the plane $\vec{r}.(\hat{i} - \hat{j} + \hat{k}) = 5$. 2

26. A purse contains 3 silver and 6 copper coins and a second purse contains 4 silver and 3 copper coins. If a coin is drawn at random from one of the two purses, find the probability that it is a silver coin. 2

SECTION - C **24 Marks**

27. Check whether the relation R in the set N of natural numbers given by

R = {(a, b) : a is divisor of b}

is reflexive, symmetric or transitive. Also determine whether R is an equivalence relation. 4

OR

*Prove that $\tan^{-1}\frac{1}{4} + \tan^{-1}\frac{2}{9} = \frac{1}{2}\sin^{-1}\left(\frac{4}{5}\right)$. 4

28. If $\tan^{-1}\left(\frac{y}{x}\right) = \log\sqrt{x^2 + y^2}$, prove that $\frac{dy}{dx} = \frac{x + y}{x - y}$. 4

OR

If $y = e^{a\cos^{-1}x}$, $-1 < x < 1$, then show that 4

$$(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a^2y = 0$$

29. Find : $\int \frac{x^3 + 1}{x^3 - x}dx$. 4

30. Solve the following differential equation:

$$(1 + e^{y/x})dy + e^{y/x}\left(1 - \frac{y}{x}\right)dx = 0\ (x \neq 0).$$ 4

31. Find the shortest distance between the lines

$$\vec{r} = 2\hat{i} - \hat{j} + \hat{k} + \lambda(3\hat{i} - 2\hat{j} + 5\hat{k})$$
$$\vec{r} = 3\hat{i} + 2\hat{j} - 4\hat{k} + \mu(4\hat{i} - \hat{j} + 3\hat{k})$$ 4

32. A cottage industry manufactures pedestal lamps and wooden shades. Both the products require machine time as well as craftsman time in the making. The number of hour(s) required for producing 1 unit of each and the corresponding profit is given in the following table:

Item	Machine Time	Craftsman time	Profit (in ₹)
Pedestal lamp	1.5 hours	3 hours	30
Wooden shades	3 hours	1 hour	20

In a day, the factory has availability of not more than 42 hours of machine time and 24 hours of craftsman time.

Assuming that all items manufactured are sold, how should the manufacturer schedule his daily production in order to maximise the profit ? Formulate it as an LPP and solve it graphically. 4

SECTION - D **24 Marks**

33. If $A = \begin{bmatrix} 5 & -1 & 4 \\ 2 & 3 & 5 \\ 5 & -2 & 6 \end{bmatrix}$, find A^{-1} and use it to solve the following system of the equations:

$$5x - y + 4z = 5$$
$$2x + 3y + 5z = 2$$
$$5x - 2y + 6z = -1$$ 6

OR

*If x, y, z are different and $\begin{vmatrix} x & x^2 & 1 + x^3 \\ y & y^2 & 1 + y^3 \\ z & z^2 & 1 + z^3 \end{vmatrix} = 0$, then using properties of determinants show that $1 + xyz = 0$. 6

*Not examinable for 2021 exam.

34. Amongst all open (from the top) right circular cylindrical boxes of volume 125π cm^3, find the dimensions of the box which has the least surface area. 6

***35.** Using integration, find the area lying above x-axis and included between the circle $x^2 + y^2 = 8x$ and inside the parabola $y^2 = 4x$. 6

OR

Using the method of integration, find the area of the triangle ABC, coordinates of whose vertices are A(2, 0), B(4, 5) and C(6, 3). 6

36. Find the probability distribution of the random variable X, which denotes the number of doublets in four throws of a pair of dice. +Hence, find the mean of the number of doublets (X). 6

*Not examinable for 2021 exam.

+Calculation of mean is not a part of Syllabus of 2020-21 examination.