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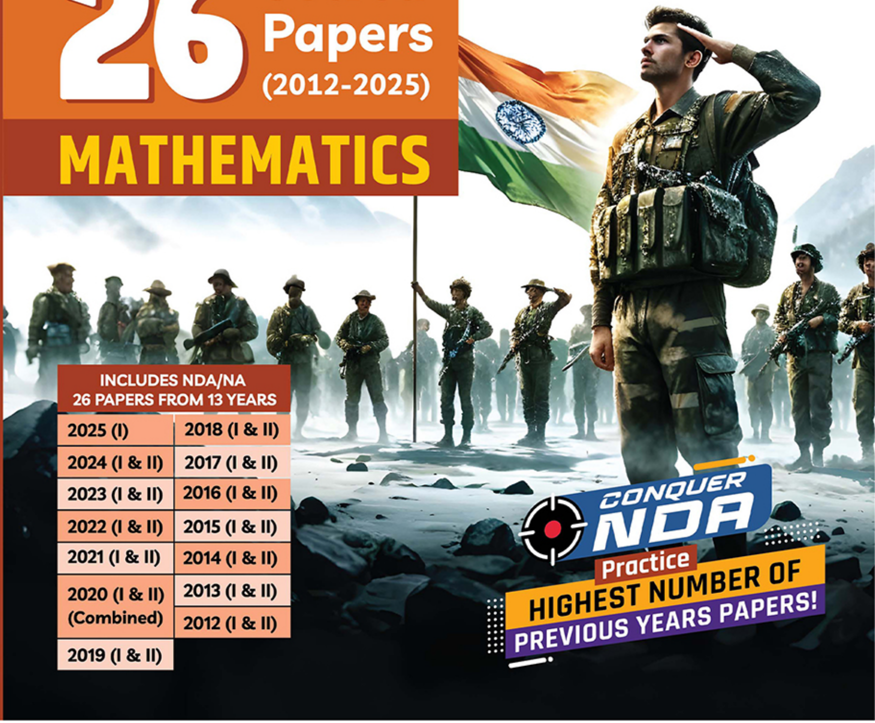
Includes analytical chart of previous years papers

26 Solved Papers (2012-2025)

MATHEMATICS



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2025 (I)	2018 (I & II)
2024 (I & II)	2017 (I & II)
2023 (I & II)	2016 (I & II)
2022 (I & II)	2015 (I & II)
2021 (I & II)	2014 (I & II)
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AGRAWAL EXAMCART

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NDA/NA MATHEMATICS

SOLVED PAPERS

2012-2025

META DATA

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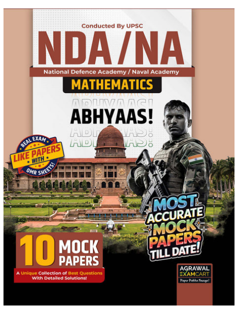

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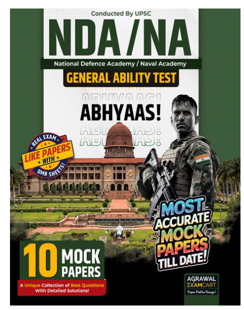

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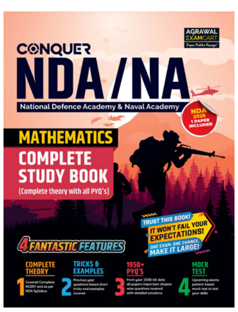

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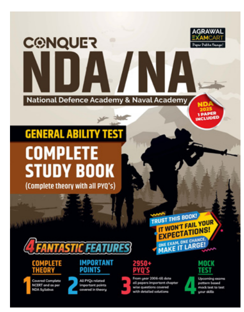

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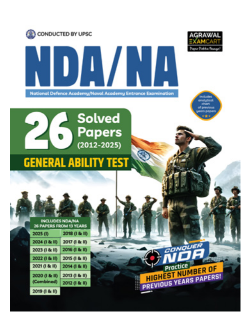
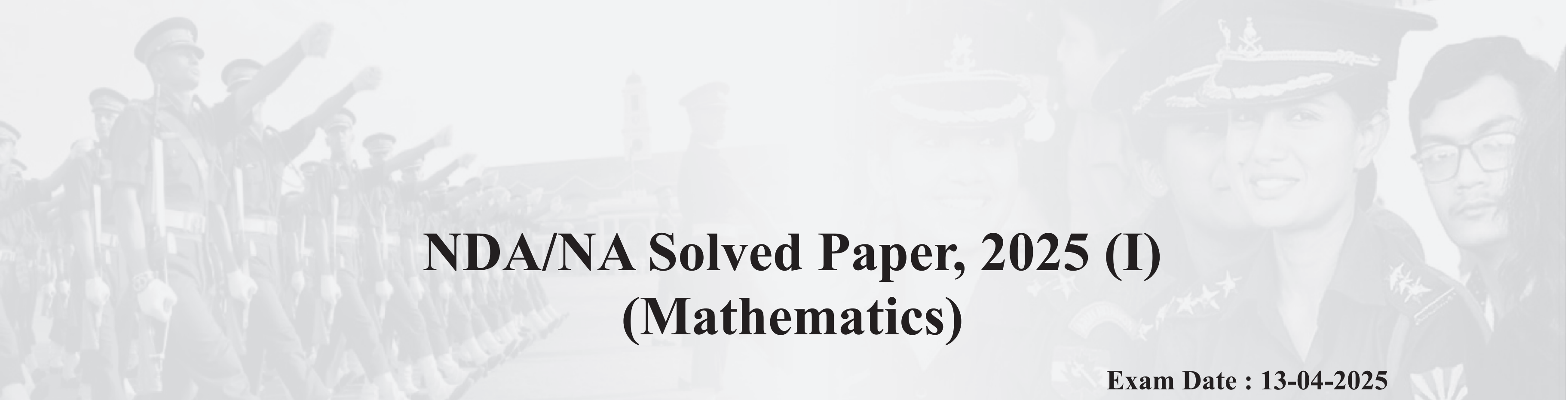


NDA/NA GAT (Solved Papers)



NDA/NA Solved Paper, 2025 (I)
(Mathematics)

Exam Date : 13-04-2025

Sets, Relations and Functions

Direction (Q. No. 1 and 2)

Consider the following for the **two** items that follow :

The function $f(x)$ satisfies $f\left(\frac{x}{y}\right) = \frac{f(x)}{f(y)}$ for all positive real values of x and y, and $f(2) = 3$.

1. What is $f(16)$ equal to ?

(A) 18 (B) 27

(C) 54 (D) 81

2. What is $f(1)f(4)$ equal to ?

(A) 4 (B) 8

(C) 9 (D) 18

Direction (Q. No. 3 and 4)

Consider the following for the **two** items that follow :

A function f is such that $f(xy) = f(x + y)$ for all real values of x and y, and $f(5) = 10$.

3. What is $f(0)$ equal to ?

(A) 0 (B) 1

(C) 5 (D) 10

4. What is $f(20) + f(-20)$ equal to ?

(A) 0 (B) 10

(C) 20 (D) 40

5. Let the function $y = (1 - \cos x)^{-1}$, where $x \neq 2n\pi$ and n is an integer.

What is the range of the function ?

(A) $[0, \infty)$ (B) $[0.5, \infty)$

(C) $[1, \infty)$ (D) $(-\infty, 0.5]$

6. Let the curve $f(x) = |x - 3|$.

What is the domain of the function $f(x)$?

(A) $(0, \infty)$ (B) $(3, \infty)$

(C) $(-\infty, \infty)$ (D) $(-\infty, \infty)\ \{3\}$

Direction (Q. No. 7 and 8)

Consider the following for the **two** items that follow :

Let $f = \{(1, 1), (2, 4), (3, 7), (4, 10)\}$.

7. If $f(x) = px + q$, then what is the value of $(p + q)$?

(A) –1 (B) 0

(C) 1 (D) 5

8. Consider the following statements :

I. f is one-one function.

II. f is onto function if the codomain is the set of natural numbers.

Which of the statements given above is/are correct ?

(A) I only (B) II only

(C) Both I and II (D) Neither I nor II

Binary Numbers

9. If $x = (1111)_2$, $y = (1001)_2$ and $z = (110)_2$, then what is $x^3 - y^3 - z^3 - 3xyz$ equal to ?

(A) $(1111001)_2$ (B) $(1001111)_2$

(C) $(1)_2$ (D) $(0)_2$

Complex Numbers

Direction (Q. No. 10 and 11)

Consider the following for the **two** items that follow :

Let $f(x) = [x^2]$, where [.] is the greatest integer function.

10. What is $\int_{\sqrt{2}}^{\sqrt{3}} f(x)\,dx$ equal to ?

(A) $\sqrt{3} - \sqrt{2}$ (B) $2(\sqrt{3} - \sqrt{2})$

(C) $3 - \sqrt{2}$ (D) 1

11. What is $\int_{\sqrt{2}}^{2} f(x)dx$ equal to ?

(A) $6 - \sqrt{3} - 2\sqrt{2}$

(B) $6 - \sqrt{3} - \sqrt{2}$

(C) $6 - \sqrt{3} + 2\sqrt{2}$

(D) $6 + \sqrt{3} - 2\sqrt{2}$

12. What is $\left(\frac{\sqrt{3}+i}{\sqrt{3}-i}\right)^3$ equal to ?

(A) –1 (B) 0

(C) 1 (D) 3

13. If $z \neq 0$ is a complex number, then what is $\text{amp}(z) + \text{amp}(\bar{z})$ equal to ?

(A) 0 (B) $\frac{\pi}{2}$

(C) π (D) 2π

Quadratic Equations and Inequalities

14. If $k < (\sqrt{2} + 1)^3 < k + 2$, where k is a natural number, then what is the value of k ?

(A) 11 (B) 13

(C) 15 (D) 17

15. If one root of the equation $x^2 - kx + k = 0$ exceeds the other by $2\sqrt{3}$, then which one of the following is a value of k ?

(A) 3 (B) 6

(C) 9 (D) 12

16. If $x + \frac{5}{y} = 4$ and $y + \frac{5}{x} = -4$, then what is $(x + y)$ equal to ?

(A) 0 (B) 1

(C) 4 (D) 5

17. If $x^2 - x + 1 = 0$, then what is $\left(x-\frac{1}{x}\right)^2 + \left(x-\frac{1}{x}\right)^4 + \left(x-\frac{1}{x}\right)^8$ equal to ?

(A) 81 (B) 85

(C) 87 (D) 90

18. What is the number of positive integer solutions of $x + y + z = 5$?

(A) 3 (B) 5

(C) 6 (D) 9

Sequence and Series

19. The sum of the first k terms of a series S is $3k^2 + 5k$. Which one of the following is correct ?

(A) The terms of S form an arithmetic progression with common difference 14.

(B) The terms of S form an arithmetic progression with common difference 6.

(C) The terms of S form a geometric progression with common ratio 10/7.

(D) The terms of S form a geometric progression with common ratio 11/4.

20. The sum of the first 8 terms of a G.P. is five times the sum of its first 4 terms. If $r \neq 1$ is the common ratio, then what is the number of possible real values of r ?

(A) One (B) Two

(C) Three (D) More than three

21. The arithmetic mean of 100 observations is 50. If 5 is subtracted from each observation and then divided by 20, then what is the new arithmetic mean ?

(A) 2.25 (B) 3.5

(C) 4.25 (D) 5.5

22. What is the arithmetic mean of 8^2, 9^2, 10^2, …, 15^2 ?

(A) 133.5 (B) 135.5

(C) 137.5 (D) 139.5

Permutations and Combinations

23. If the number of selections of r as well as $(n+r)$ things from $5n$ different things are equal, then what is the value of r ?

(A) n (B) $2n$

(C) $3n$ (D) $4n$

24. What is the number of selections of at most 3 things from 6 different things ?

(A) 20 (B) 22

(C) 41 (D) 42

25. How many 7-letter words (with or without meaning) can be constructed using all the letters of the word CAPITAL so that all consonants come together in each word ?

(A) 360 (B) 300

(C) 288 (D) 240

26. How many sides are there in a polygon which has 20 diagonals ?

(A) 6 (B) 7

(C) 8 (D) 10

27. In how many ways can the letters of its word DELHI be arranged keeping the positions of vowels and consonants unchanged ?

(A) 6 (B) 9

(C) 12 (D) 24

Direction (Q. No. 28 and 29)

Consider the following for the **two** items that follow :

A committee of 6 members is formed from a group of 7 gentlemen and 4 ladies:

28. What is the probability that the committee includes exactly 3 gentlemen ?

(A) $\frac{10}{33}$ (B) $\frac{30}{77}$

(C) $\frac{100}{231}$ (D) $\frac{5}{11}$

29. What is the probability that the committee includes at least 2 ladies ?

(A) $\frac{41}{66}$ (B) $\frac{47}{66}$

(C) $\frac{49}{66}$ (D) $\frac{53}{66}$

Binomial Theorem

30. If the sum of binomial coefficients in the expansion of $(x+y)^n$ is 256, then the greatest binomial coefficient occurs in which one of the following terms ?

(A) Third (B) Fourth

(C) Fifth (D) Ninth

31. Let X be a random variable following binomial distribution whose mean and variance are 200 and 160 respectively. What is the value of the number of trials (n) ?

(A) 500 (B) 1000

(C) 1500 (D) 2000

32. What is the number of rational terms the expansion of $(3^{\frac{1}{2}}+5^{\frac{1}{4}})^{12}$?

(A) 2 (B) 3

(C) 4 (D) 6

Trigonometry

Direction (Q. No. 33 to 35)

Consider the following for the **three** items that follow :

Let $p = \tan 2\alpha - \tan\alpha$ and $q = \cot\alpha - \cot 2\alpha$.

33. What is $\left(\frac{p}{q}\right)$ equal to ?

(A) $-\tan\alpha \cdot \tan 2\alpha$

(B) $-\cot\alpha \cdot \cot 2\alpha$

(C) $\tan\alpha \cdot \tan 2\alpha$

(D) $\cot\alpha \cdot \cot 2\alpha$

34. What is $(p+q)$ equal to ?

(A) $\sec 4\alpha$

(B) $\operatorname{cosec} 4\alpha$

(C) $2\sec 4\alpha$

(D) $2\operatorname{cosec} 4\alpha$

35. What is $\tan^2\alpha$ equal to?

(A) $(pq)/(p+q)$

(B) $(p+2q)/p$

(C) $p/(p+2q)$

(D) $p/(2p+q)$

Direction (Q. No. 36 and 37)

Consider the following for the **two** items that follow :

Let $2\sin\alpha + \cos\alpha = 2$, where $0 < \alpha < 90^\circ$.

36. What is $\tan\alpha$ equal to ?

(A) $\frac{1}{2}$ (B) 1

(C) $\frac{3}{4}$ (D) 2

37. What is $2\sin 2\alpha + \cos 2\alpha$ equal to ?

(A) $\frac{11}{10}$ (B) $\frac{11}{5}$

(C) $\frac{12}{5}$ (D) $\frac{13}{5}$

38. If 5^{th}, 7^{th} and 13^{th} terms of an A.P. are in G.P., then what is the ratio of its first term to its common difference ?

(A) –3 (B) –2

(C) 2 (D) 3

39. If p, 1, q are in A.P. and p, 2, q are in G.P. then which of the following statements is/are correct ?

I. p, 4, q are in H.P.

II. $\left(\frac{1}{p}\right), \frac{1}{4}, \left(\frac{1}{q}\right)$ are in A.P.

Select the answer using the code given below.

(A) I only (B) II only

(C) Both I and II (D) Neither I nor II

Direction (Q. No. 40 to 42)

Consider the following for the **three** items that follow :

Let $p = \sin 35^\circ$, $q = \sin 25^\circ$ and $r = \sin(-95^\circ)$.

40. What is $(p+q+r)$ equal to ?

(A) –1 (B) 0

(C) $2\sin 5^\circ$ (D) $2\cos 5^\circ$

41. What is $(pq + qr + rp)$ equal to ?

(A) $-\frac{3}{4}$ (B) 0

(C) $\frac{1}{4}$ (D) $\frac{3}{4}$

42. What is $(p^2 + q^2 + r^2)$ equal to ?

(A) $\frac{1}{2}$ (B) 1

(C) $\frac{3}{2}$ (D) 2

Direction (Q. No. 43 and 44)

Consider the following for the **two** items that follow :

Let $p = |\sin\alpha - \sin(\alpha - 90^\circ)|$.

43. What is the minimum value of p ?

(A) 0 (B) $\frac{1}{2}$

(C) $\frac{1}{\sqrt{2}}$ (D) 1

44. What is the maximum value of p ?

(A) 1 (B) $\sqrt{2}$

(C) $\sqrt{3}$ (D) 2

Properties of a Triangle

Direction (Q. No. 45 and 46)

Consider the following for the **two** items that follow :

In a triangle ABC, two sides BC and CA are in the ratio 2 : 1 and their opposite corresponding angles are in the ratio 3 : 1.

45. One of the angles of the triangle is :

(A) 15º (B) 30º

(C) 45º (D) 75º

46. Consider the following statements :

I. The triangle is right-angled.

II. One of the sides of the triangle is 3 times the other.

III. The angles A, C and B of the triangle are in A.P.

Which of the statements given above is/are correct ?

(A) 1 only

(B) II and III only

(C) I and Ill only

(D) 1, II and III

Direction (Q. No. 47 to 49)

Consider the following for the **three** items that follow :

The sides of a triangle ABC are AB = 3 cm, BC = 5 cm and CA = 7 cm.

47. Consider the following statements :

I. The triangle is obtuse-angled triangle.

II. The sum of acute angles of the triangle is also acute.

Which of the statements given above is/are correct ?

(A) I only (B) II only

(C) Both I and II (D) Neither I nor II

48. What is $\angle B$ equal to ?

(A) 60º (B) 105º

(C) 120º (D) 150º

49. What is the area of the triangle ?

(A) $\frac{15\sqrt{3}}{4}$ square cm

(B) $\frac{15\sqrt{3}}{2}$ square cm

(C) $15\sqrt{3}$ square cm

(D) $30\sqrt{3}$ square cm

Direction (Q. No. 50 and 51)

Consider the following for the **two** items that follow :

Consider the following for the **two** items that follow :

Let ABC be a triangle right-angled at B and AB + AC = 3 units.

50. What is $\angle A$ equal to if the area of the triangle is maximum ?

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$

(C) $\frac{\pi}{3}$ (D) $\frac{5\pi}{12}$

51. What is the maximum area of the triangle ?

(A) $\frac{\sqrt{3}}{2}$ square unit

(B) $\sqrt{3}$ square units

(C) $\frac{\sqrt{6}}{2}$ square units

(D) $\sqrt{6}$ square units

Height and Distance

52. A man at M, standing 100 m away from the base (P) of a chimney of height 50 m, observes the angle of elevation of the highest point (Q) of the smoke to be 45°. The highest point of the chimney is at R. Further P, R and Q are in a straight line and the straight line is perpendicular to PM. What is the angle RMQ equal to?

(A) $\tan^{-1}\left(\frac{1}{2}\right)$ (B) $\tan^{-1}\left(\frac{1}{3}\right)$

(C) $\tan^{-1}\left(\frac{2}{3}\right)$ (D) $\tan^{-1}\left(\frac{3}{4}\right)$

Direction (Q. No. 53 and 54)

Consider the following for the **two** items that follow :

The top (M) of a tower is observed from three points P, Q and R lying in a horizontal straight line which passes directly along the foot (N) of the tower. The angles of elevations of M from P, Q and R are 30º, 45º and 60º respectively. Let PQ = a and QR = b.

53. What is PN equal to?

(A) $\left(\frac{3-\sqrt{3}}{2}\right)a$ (B) $\left(\frac{3+\sqrt{3}}{2}\right)a$

(C) $\left(\frac{3-\sqrt{3}}{4}\right)a$ (D) $\left(\frac{3+\sqrt{3}}{4}\right)a$

54. What is MN equal to?

(A) $\left(\frac{3+\sqrt{3}}{2}\right)b$ (B) $\left(\frac{3-\sqrt{3}}{2}\right)b$

(C) $\left(\frac{3-\sqrt{3}}{4}\right)b$ (D) $\left(\frac{3+\sqrt{3}}{4}\right)b$

Inverse Trigonometric Functions

55. If k is a root of $x^2 - 4x + 1 = 0$, then what is $\tan^{-1}k + \tan^{-1}\frac{1}{k}$ equal to ?

(A) $-\frac{\pi}{2}$ (B) 0

(C) $\frac{\pi}{4}$ (D) $\frac{\pi}{2}$

56. If $\tan^{-1}k + \tan^{-1}\frac{1}{2} = \frac{\pi}{4}$, then what is the value of k ?

(A) 1 (B) $\frac{1}{2}$

(C) $\frac{1}{3}$ (D) $\frac{1}{4}$

Direction (Q. No. 57 and 58)

Consider the following for the **two** items that follow :

Let $y = \sin^{-1}\left(x - \frac{4x^3}{27}\right)$.

57. What is y equal to ?

(A) $\sin^{-1} x$ (B) $\sin^{-1}\left(\frac{x}{3}\right)$

(C) $3\sin^{-1} x$ (D) $3\sin^{-1}\left(\frac{x}{3}\right)$

58. What is $\frac{dy}{dx}$ equal to ?

(A) $\frac{1}{\sqrt{9-x^2}}$ (B) $\frac{1}{\sqrt{3-x^2}}$

(C) $\frac{3}{\sqrt{9-x^2}}$ (D) $\frac{9}{\sqrt{9-x^2}}$

Matrices

59. If

$$[x\,1\,1]\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\\ 7 & 8 & 9\end{bmatrix}\begin{bmatrix}1\\ 1\\ x\end{bmatrix} = [45]$$

then which one of the following is a value of x ?

(A) –2 (B) –1

(C) 0 (D) 1

60. If

$$A = \begin{bmatrix}y & z & x\\ z & x & y\\ x & y & z\end{bmatrix}$$

where x, y, z are integers, is an orthogonal matrix, then what is the value of $x^2 + y^2 + z^2$?

(A) 0 (B) 1

(C) 4 (D) 14

61. If $f(\theta) = \begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$

then what is $\{f(\pi)\}^2$ equal to ?

(A) $\begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix}$ (B) $\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}$

(C) $\begin{bmatrix}-1 & 0\\ 0 & 1\end{bmatrix}$ (D) $\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

62. If

$$A = \begin{bmatrix}1 & 2 & 2\\ 2 & 1 & 2\\ 2 & 2 & 1\end{bmatrix}$$

then what is $A^2 - 4A$ equal to ?

(A) $-5I_3$ (B) $-I_3$

(C) I_3 (D) $5I_3$

where I_3 is the identity matrix of order 3.

63. If

$$A = \begin{bmatrix}x & y & z\\ y & z & x\\ z & x & y\end{bmatrix}$$

where x, y, z are integers, is an orthogonal matrix, then what is A^2 equal to ?

(A) Null matrix

(B) Identity matrix

(C) A

(D) –A

Determinants

64. Consider the following in respect of a non-singular matrix M :

I. $|M^2| = |M|^2$

II. $|M| = |M^{-1}|$

III. $|M| = |M^T|$

How many of the above are correct ?

(A) None (B) One

(C) Two (D) All three

65. If

$$\Delta = \begin{vmatrix}a & b & c\\ d & e & f\\ g & h & i\end{vmatrix}$$

and A, B, C, D, G are the cofactors of the elements a, b, c, d, g respectively, then what is $bB + cC - dD - gG$ equal to ?

(A) 0 (B) 1

(C) Δ (D) $-\Delta$

66. Consider the following statements in respect of the determinant

$$\Delta = \begin{vmatrix}k(k+2) & 2k+1 & 1\\ 2k+1 & k+2 & 1\\ 3 & 3 & 1\end{vmatrix}$$

I. Δ is positive if $k > 0$.

II. Δ is negative if $k < 0$.

III. Δ is zero if $k = 0$.

How many of the statements given above are correct ?

(A) None (B) One

(C) Two (D) All three

67. If

$$\begin{bmatrix}2 & 3+i & -1\\ 3-i & 0 & i-1\\ -1 & -1-i & 1\end{bmatrix} = A + iB$$

where $i = \sqrt{-1}$, then what is A + B equal to ?

(A) – 10 (B) – 6

(C) 0 (D) 6

68. If $A^2 + B^2 + C^2 = 0$, then what is the value of the following ?

$$\begin{bmatrix}1 & \cos C & \cos B\\ \cos C & 1 & \cos A\\ \cos B & \cos A & 1\end{bmatrix}$$

(A) –1 (B) 0

(C) 1 (D) 2

69. If ω is a non-real cube root of unity, then what is a root of the following equation ?

$$\begin{bmatrix}x+1 & \omega & \omega^2\\ \omega & x+\omega^2 & 1\\ \omega^2 & 1 & x+\omega\end{bmatrix} = 0$$

(A) $x = 0$ (B) $x = 1$

(C) $x = \omega$ (D) $x = \omega^2$

Cartesian Co-ordinates System and Straight Lines

70. Under what condition will the lines $m^2x + ny - 1 = 0$ and $n^2x - my + 2 = 0$ be perpendicular ?

(A) $mn - 1 = 0$ (B) $mn + 1 = 0$

(C) $m + n = 0$ (D) $m - n = 0$

71. If p and q are real numbers between 0 and 1 such that the points $(p, 1)$,$(1, q)$ and $(0, 0)$ form an equilateral triangle, then what is $(p + q)$ equal to?

(A) $\sqrt{2}$ (B) $\sqrt{2}-1$

(C) $2-\sqrt{3}$ (D) $4-2\sqrt{3}$

72. The vertices of a triangle are A(1, 1), B(0, 0) and C(2, 0). The angular bisectors of the triangle meet at P. What are the coordinates of P ?

(A) $(1, \sqrt{2} - 1)$ (B) $(1, \sqrt{3} - 1)$

(C) $\left(1, \frac{1}{2}\right)$ (D) $\left(\frac{1}{2}, \sqrt{2}-1\right)$

73. Let A(3, –1) and B(1, 1) be the end points of line segment AB. Let P be the middle point of the line segment AB. Let Q be the point situated at a distance $\sqrt{2}$ units, from P on the perpendicular bisector line of AB. What are the possible coordinates of Q ?

(A) (2, 1) (B) (3, 1)
(C) (2, 2) (D) (1, 3)

74. ABC is an equilateral triangle and AD is the altitude on BC. If the coordinates of A are (1, 2) and that of D are (–2, 6), then what is the equation of BC ?

(A) $3x + 4y - 18 = 0$
(B) $4x + 3y - 1 = 0$
(C) $4x - 3y + 26 = 0$
(D) $3x - 4y + 30 = 0$

75. What is the equation of the circle whose diameter is 10 cm and the equations of two of its diameters are $x + y = 0$ and $x - y = 0$?

(A) $x^2 + y^2 = 1$
(B) $x^2 + y^2 = 25$
(C) $x^2 + y^2 = 100$
(D) $x^2 + y^2 - 2x - 2y - 23 = 0$

76. A square is inscribed in a circle $x^2 + y^2 + 2x + 2y + 1 = 0$ and its sides are parallel to coordinate axes. Which one of the following is a vertex of the square ?

(A) (–2, 2)
(B) (–2, –2)
(C) $\left(-1+\frac{1}{\sqrt{2}}, -1-\frac{1}{\sqrt{2}}\right)$
(D) None of the above

77. The slope of the tangent to the curve $y = f(x)$ at $(x, f(x))$ is 4 for every real number x and the curve passes through the origin.

What is the nature of the curve ?

(A) A straight line passing through (1, 4)
(B) A straight line passing through (–1, 4)
(C) A parabola with vertex at origin and focus at (2, 0)
(D) A parabola with vertex at origin and focus at (1, 0)

Conic Sections

78. A tangent to the parabola $y^2 = 4x$ is inclined at an angle 45º with the positive direction of x-axis. What is the point of contact of the tangent and the parabola ?

(A) (1, 1) (B) $(2, 2\sqrt{2})$
(C) $\left(\frac{1}{2}, \frac{1}{\sqrt{2}}\right)$ (D) (1, 2)

79. What is the distance between the two foci of the hyperbola $25x^2 - 75y^2 = 225$?

(A) $2\sqrt{3}$ units
(B) $4\sqrt{3}$ units
(C) $\sqrt{6}$ units
(D) $2\sqrt{6}$ units

80. If any point on an ellipse is (3sinα, 5cosα), then what is the eccentricity of the ellipse ?

(A) $\frac{4}{3}$ (B) $\frac{4}{5}$
(C) $\frac{3}{4}$ (D) $\frac{1}{2}$

Limits, Continuity and Differentiability

Direction (Q. No. 81 and 82)

Consider the following for the **two** items that follow :

Let the function $f(x) = x^2 + 9$

81. What is $\lim_{x\to 0}\frac{\sqrt{f(x)}-3}{\sqrt{f(x)+7}-4}$ equal to ?

(A) $\frac{2}{3}$ (B) 1
(C) $\frac{4}{3}$ (D) 2

82. Consider the following statements :

I. $f(x)$ is an increasing function.
II. $f(x)$ has local maximum at $x = 0$

Which of the statements given above is/are correct?

(A) I only
(B) II only
(C) Both 1 and II
(D) Neither I nor II

Direction (Q. No. 83 and 84)

Consider the following for the **two** items that follow :

Let the function $f(x) = \sin[x]$, where $[\cdot]$ is the greatest integer function and $g(x) = |x|$

83. What is $\lim_{x\to 0}\{f(x)g(x)\}$ equal to ?

(A) –1
(B) 0
(C) 1
(D) Limit does not exist

84. What is $\lim_{x\to 0}\frac{f(x)}{g(x)}$ equal to ?

(A) –sin 1
(B) sin 1
(C) 0
(D) Limit does not exist

85. Let the function $f(x) = x^2 - 1$.

What is $\lim_{x\to 1}\{f\circ f(x)\}$ equal to ?

(A) –1 (B) 0
(C) 1 (D) 2

Direction (Q. No. 86 and 87)

Consider the following for the **two** items that follow :

Let $f(x)$ $f(x)=\begin{cases}x^3, & x^2<1\\ x^2, & x^2\geq 1\end{cases}$

86. What is $\lim_{x\to 0}f'(x)$ equal to ?

(A) 2
(B) 1
(C) 0
(D) Limit does not exist

87. Consider the following statements :

I. The function is continuous at $x = -1$.
II. The function is differentiable as $x = 1$.

Which of the statements given above is/are correct ?

(A) I only (B) II only
(C) Both I and II (D) Neither I nor II

Differentiation

Direction (Q. No. 88 and 89)

Consider the following for the **two** items that follow :

Let $(x + y)^{p+q} = x^p y^q$, where p, q are positive integers.

88. The derivative of y with respect to x :

(A) depends on p only
(B) depends on q only
(C) depends on both p and q
(D) is independent of both p and q

89. If $p + q = 10$, then what is $\frac{dy}{dx}$ equal to ?

(A) $\frac{y}{x}$ (B) xy
(C) $x^{10}y^{10}$ (D) $\left(\frac{y}{x}\right)^{10}$

Indefinite Integrals

90. Let the function $y = (1 - \text{cis}\, x)^{-1}$, where $x \neq 2n\pi$ and n is an integer.

What is $\int y dx$ equal to ?

(A) $-\tan\left(\frac{x}{2}\right) + c$ (B) $-\cot\left(\frac{x}{2}\right) + c$

(C) $\tan\left(\frac{x}{2}\right) + c$ (D) $\cot\left(\frac{x}{2}\right) + c$

where c is the constant of integration.

91. The slope of the tangent to the curve $y = f(x)$ at $(x, f(x))$ is 4 for every real number x and the curve passes through the origin.
What is the area bounded by the curve, the x-axis and the line $x = 4$?

(A) 8 square units (B) 16 square units

(C) 32 square units (D) 64 square units

Application of Integrals

92. Let the curve $f(x) = |x - 3|$.
What is the area bounded by the curve $f(x)$ and $y = 3$?

(A) 3 square units (B) 4.5 square units

(C) 7.5 square units (D) 9 square units

93. Let the function $f(x) = x^2 - 1$.
What is the area bounded by the function $f(x)$ and the x-axis ?

(A) $\frac{1}{3}$ square units (B) $\frac{2}{3}$ square units

(C) $\frac{4}{3}$ square units (D) 2 square units

Differential Equations

Direction (Q. No. 94 and 95)

Consider the following for the **four** items that follow :

Let $x = \sec\theta - \cos\theta$ and $y = \sec^4\theta - \cos^4\theta$.

94. What is $\left(\frac{dy}{dx}\right)^2$ equal to ?

(A) $\frac{4(y^2+4)}{(x^2+4)}$

(B) $\frac{4(y^2-4)}{(x^2-4)}$

(C) $\frac{16(y^2+4)}{(x^2+4)}$

(D) $\frac{16(y^2-4)}{(x^2-4)}$

95. What is $\left(\frac{x^2+4}{y^2+4}\right)\frac{dy}{dx}\left[(x^2+4)\frac{d^2y}{dx^2} - 16y\right]$ equal to ?

(A) $16x$ (B) $16y$

(C) $-16x$ (D) $-16y$

Vector Algebra

96. A line makes angles α, β and γ with the positive directions of the coordinate axes.
If $\vec{a} = (\sin^2\alpha)\hat{i} + (\sin^2\beta)\hat{j} + (\sin^2\gamma)\hat{k}$ and $\vec{b} = \hat{i} + \hat{j} + \hat{k}$, then what is $\vec{a}\cdot\vec{b}$ equal to ?

(A) -2 (B) -1

(C) 1 (D) 2

97. Consider the following statements in respect of a vector $\vec{d} = (\vec{a} \times \vec{b}) \times \vec{c}$;

I. $\vec{d}$ is coplanar with $\vec{a}$ and $\vec{b}$.

II. $\vec{d}$ is perpendicular to $\vec{c}$.

Which of the statements given above is/are correct ?

(A) I only (B) II only

(C) Both I and II (D) Neither 1 nor II

98. The position vectors of three points A, B and C are $\vec{a}$, $\vec{b}$ and $\vec{c}$ respectively such that $3\vec{a} - 4\vec{b} + \vec{c} = \vec{0}$. What is AB: BC equal to ?

(A) 3 : 1 (B) 1 : 3

(C) 3 : 4 (D) 1 : 4

99. The position vectors of three points A, B and C are $\vec{a}$, $\vec{b}$ and $\vec{c}$ respectively, where $\vec{c} = (\cos^2\theta)\vec{a} + (\sin^2\theta)\vec{b}$. What is $(\vec{a}\times\vec{b}) + (\vec{b}\times\vec{c}) + (\vec{c}\times\vec{a})$ equal to ?

(A) $\vec{0}$ (B) $2\vec{c}$

(C) $3\vec{c}$ (D) Unit vector

100. Let $\vec{a}, \vec{b}, (\vec{a}\times\vec{b})$ be unit vectors. What is $(\vec{a}\cdot\vec{b})$ equal to ?

(A) 0 (B) $\frac{1}{2}$

(C) 1 (D) 3

3D Geometry

101. If a line in 3 dimensions makes angles α, β and γ with the positive directions of the coordinate axes, then what is $\cos(\alpha + \beta)\cos(\alpha + \beta)$ equal to ?

(A) $\cos^2\gamma$ (B) $-\cos^2\gamma$

(C) $\sin^2\gamma$ (D) $-\sin^2\gamma$

102. A(1, 2, –1), B(2, 5, –2) and C(4, 4, –3) are three vertices of a rectangle. What is the area of the rectangle ?

(A) 8 square units

(B) 9 square units

(C) $\sqrt{66}$ square units

(D) $\sqrt{68}$ square units

103. ABC is a triangle right-angled at B. If A(k, 1, –1), B(2k, 0, 2) and C(2 + 2k, k, 1) are the vertices of the triangle, then what is the value of k ?

(A) –3 (B) –1

(C) 1 (D) 3

104. If a line

$$\frac{x+1}{p} = \frac{y-1}{q} = \frac{z-2}{r}$$

where $p = 2q = 3r$, makes an angle θ with the positive direction of y-axis, then what is $\cos 2\theta$ equal to ?

(A) $-\frac{31}{49}$ (B) $-\frac{37}{49}$

(C) $\frac{31}{49}$ (D) $\frac{37}{49}$

105. What is the equation of the plane passing through the point (1, 1, 1) and perpendicular to the line whose direction ratios are (3, 2, 1) ?

(A) $x + 2y + 3z = 6$

(B) $3x + 2y + z = 6$

(C) $x + y + z = 3$

(D) $3x + 2y + z = 0$

106. The standard deviation of 100 observations is 10. If 5 is added to each observation and then divided by 20, then what will be the new standard deviation ?

(A) 0.25 (B) 0.5

(C) 0.75 (D) 1.00

Statistics

Direction (Q. No. 107 to 110)

Consider the following for the **four** items that follow :

The frequency distribution of height of students of a class is given below :

Height (in cm)	Number of Students
160-162	12
162-164	15
164-166	24
166-168	13

107. What is the total number of students whose height is less than or equal to 165 cm ?

(A) 15 (B) 39

(C) 51 (D) None of these

108. What is the median height of the class ?

(A) 162.41 cm (B) 163.41 cm

(C) 164.41 cm (D) 165.41 cm

109. The height which occurs most frequently in the class is :

(A) 163.5 cm (B) 163.9 cm

(C) 164.5 cm (D) 1649. cm

110. The most appropriate graphical representation of the given frequency distribution is :

(A) bar chart

(B) percentage bar chart

(C) histogram

(D) Pie chart

Direction (Q. No. 111 and 112)

Consider the following for the **two** items that follow :

The sum and the sum of squares of the observations corresponding to length X (in cm) and weight Y (in gm) of 50 tropical tubers are given as $\Sigma X = 200$, $\Sigma Y = 250$, $\Sigma X^2 = 900$ and $\Sigma Y^2 = 1400$.

111. Which one of the following is correct?

(A) Variance (X) > Variance (Y)

(B) Variance (X) < Variance (Y)

(C) Variance (X) = Variance (Y)

(D) Cannot be determined from the given data

112. Which one of the following statements is correct ?

(A) Coefficient of variation of X is strictly more than coefficient of variation of Y.

(B) Coefficient of variation of X is strictly less than coefficient of variation of Y.

(C) Coefficient of variation of X is same as coefficient of variation of Y.

(D) Coefficient of variation cannot be determined from the given data.

Probability

Direction (Q. No. 113 and 114)

Consider the following for the **two** items that follow :

The probabilities that A, B and C become managers are $\frac{3}{10}$, $\frac{1}{2}$ and $\frac{4}{5}$ respectively.

The probabilities that bonus scheme will be introduced if A, B and C become managers are $\frac{4}{9}$, $\frac{2}{9}$ and $\frac{1}{3}$ respectively.

113. What is the probability that the bonus scheme will be introduced ?

(A) $\frac{17}{45}$ (B) $\frac{19}{45}$

(C) $\frac{23}{45}$ (D) $\frac{26}{45}$

114. If the bonus scheme has been introduced, then what is the probability that the manager appointed was B ?

(A) $\frac{5}{23}$ (B) $\frac{6}{23}$

(C) $\frac{7}{23}$ (D) $\frac{8}{23}$

115. If $P(A) = \frac{1}{3}$, $P(B) = \frac{1}{2}$ and $P(A \cap B) = \frac{1}{4}$, then what is value of $P(B|A^c)$?

(A) $\frac{1}{8}$ (B) $\frac{3}{8}$

(C) $\frac{5}{8}$ (D) $\frac{7}{8}$

116. If $P(A) = \frac{1}{3}$, $P(B) = \frac{1}{2}$ and $P(A \cap B) = \frac{1}{4}$, then what is the value of $P(A^c \cap B^c)$?

(A) $\frac{1}{4}$ (B) $\frac{5}{12}$

(C) $\frac{7}{12}$ (D) $\frac{11}{12}$

117. If two fair dice are tossed, then what is the probability that the sum of the numbers on the faces of the dice is strictly greater than 7 ?

(A) $\frac{1}{3}$

(B) $\frac{5}{12}$

(C) $\frac{7}{12}$

(D) $\frac{3}{4}$

118. The probability of a man hitting a target is $\frac{1}{5}$. If the man fires 7 times, then what is the probability that he hits the target at least twice ?

(A) $1-\left(\frac{3}{5}\right)\left(\frac{4}{5}\right)^6$

(B) $1-\left(\frac{3}{5}\right)\left(\frac{4}{5}\right)^7$

(C) $1-\left(\frac{11}{5}\right)\left(\frac{4}{5}\right)^6$

(D) $1-\left(\frac{11}{5}\right)\left(\frac{4}{5}\right)^7$

Direction (Q. No. 119 and 120)

Consider the following for the **two** items that follow :

Let X be is random variable following binomial distribution with parameters $n = 6$ and $P = k$. further, $9P(X = 4) = P(X = 2)$.

119. What is the value of k ?

(A) $\frac{1}{2}$

(B) $\frac{1}{3}$

(C) $\frac{1}{4}$

(D) $\frac{1}{5}$

120. What is the value of $P(X = 3)$?

(A) $\frac{135}{1024}$

(B) $\frac{5}{128}$

(C) $\frac{45}{1024}$

(D) $\frac{70}{1024}$

Solutions

1. (D) If $x = y = 1$

then $f\left(\frac{1}{1}\right) = \frac{f(1)}{f(1)}$

$f(1) = 1$

If $x = 4, y = 2$

then $f(2) = \frac{f(4)}{f(2)}$

$\Rightarrow$ $f(4) = [f(2)]^2 = 3^2 = 9$

If $x = 8$ and $y = 2$

$f(4) = \frac{f(8)}{f(2)}$

$f(8) = f(4) \times f(2)$

$= 9 \times 3 = 27$

Similarly

$f(16) = 81$

2. (C) $f(1)\,f(4) = 1 \times 9 = 9$

3. (D) If $x = 0$

then $f(0) = f(0 + y)$

$f(0) = f(y)$

Here y is constant

Hence,

$f(5) = f(0)$

$f(0) = 10$

4. (C) Now, $f(20) = f(0) = 10$

$f(-20) = f(0) = 10$

$\therefore f(20) + f(-20) = 10 + 10 = 20$

5. (B) $y = (1 - \cos x)^{-1}$

Here,

$-1 \le \cos x \le +1$

$+1 \ge -\cos x \ge -1$

$2 \ge 1 - \cos x \ge 0$

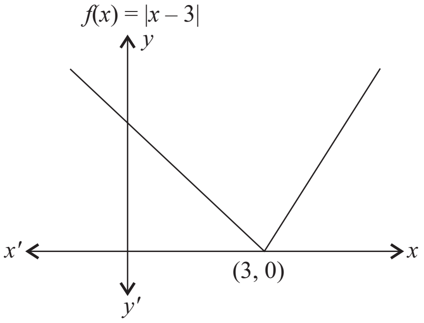
$\frac{1}{2} \le \frac{1}{1-\cos x} \le \frac{1}{0}$

$0.5 \le \frac{1}{1-\cos x} \le 0$

Hence, Range = [0.5, ∞]

6. (C) Given that

$f(x) = |x - 3|$



Hence, domain = (– ∞, ∞)

7. (C) Given that

$f = \{(1, 1), (2, 4), (3, 7), (4, 10)\}$

and $f(x) = px + q$ ...(i)

when $x = 1$ then $f(x) = 1$

from eqn. (i)

$1 = p \times 1 + q$

$p + q = 1$ ...(ii)

when $x = 2$ then $f(x) = 4$

from eqn. (i)

$4 = p \times 2 + q$

$2p + q = 4$ ...(ii)

on solving eqn. (ii) and (iii)

$p = 3\ q = -2$

Hence $p + q = 1$

8. (A) Given that $f = \{(1, 1), (2, 4), (3, 7), (4, 10)\}$

Hence, x has only one and unique image in $f(x)$

So, given function is one-one function.

Now,

Domain of $f(x) = \{1, 2, 3, 4\}$

Range of $f(x) = \{1, 4, 7, 10\}$

Given codomain = {1, 2, 3, 4 .... ∞}

Given function is not onto function because Range and codomain are not equal.

Hence, only statement I is correct only.

9. (D) $x = (1111)_2$

$= 1 \times 2^0 + 1 \times 2^1 + 1 \times 2^2 + 1 \times 2^3$

$= 1 + 2 + 4 + 8$

$= 15$

$y = (1001)_2$

$= 1 \times 2^0 + 0 \times 2^1 + 0 \times 2^2 + 1 \times 2^3$

$= 1 + 8 = 9$

$z = (110)_2$

$= 0 \times 2^0 + 1 \times 2^1 + 1 \times 2^2$

$= 0 + 2 + 4 = 6$

$x^3 - y^3 - z^3 - 3xyz = (15)^3 - 9^3 - 6^3 - 3 \times 15 \times 9 \times 6$

$= 3375 - 729 - 216 - 2430$

$= 0$

10. (B) $\int_{\sqrt{2}}^{\sqrt{3}} f(x)\, dx$

$= \int_{\sqrt{2}}^{\sqrt{3}} [x^2]\, dx$

$= [x]_{\sqrt{2}}^{\sqrt{3}}$

$[\because (\sqrt{2})^2 < x^2 < (\sqrt{3})^2$

$\therefore 2 < x^2 < 3$

Hence, there is no greatest integer function]

$= 2\left(\sqrt{3} - \sqrt{2}\right)$

11. (A) $\int_{\sqrt{2}}^{2} f(x)dx$

$\int_{\sqrt{2}}^{\sqrt{3}} [x^2]dx + \int_{\sqrt{3}}^{2} [x^2]dx$

$= 2[x]_{\sqrt{2}}^{\sqrt{3}} + 3[x]_{\sqrt{3}}^{2}$

$= 2(\sqrt{3} - \sqrt{2}) + 3(2 - \sqrt{3})$

$= 2\sqrt{3} - 2\sqrt{2} + 6 - 3\sqrt{3}$

$= 6 - \sqrt{3} - 2\sqrt{2}$

12. (A) $\left(\frac{\sqrt{3}+i}{\sqrt{3}-i}\right)^3$

On rationalizing the denominator

$= \left(\frac{\sqrt{3}+i}{\sqrt{3}-i} \times \frac{\sqrt{3}+i}{\sqrt{3}+i}\right)^3$

$= \left(\frac{3 + i^2 + 2\sqrt{3}i}{3 - i^2}\right)^3$

$= \left(\frac{3 - 1 + 2\sqrt{3}i}{3+1}\right)^3$ $[\because i^2 = -1]$

$= \left(\frac{2 + 2\sqrt{3}i}{4}\right)^3$

$= \left(\frac{1 + \sqrt{3}i}{2}\right)^3$

$= \frac{1 + 3\sqrt{3}\, i^3 + 3 \times \sqrt{3}\, i\, (1 + \sqrt{3}\, i)}{8}$

$= \frac{1 - 3\sqrt{3}\, i + 3\sqrt{3}\, i + 9\, i^2}{8}$

$= \frac{1 - 9 \times 1}{8}$

$= -\frac{8}{8} = -1$

13. (A) Let a complex number $z = x + iy$ then its conjugate $(\bar{z}) = x - iy$

$\therefore$ amp(z) + amp$(\bar{z})$ =

$\tan^{-1}\frac{y}{x} + \tan^{-1}\left(\frac{-y}{x}\right)$

$= \tan^{-1}\frac{y}{x} - \tan^{-1}\frac{y}{x} = 0$

$[\because \tan^{-1}(-\theta) = -\tan^{-1}\theta]$

14. (B) Given that

$k < (\sqrt{2}+1)^3 < k+2$

$k < 2\sqrt{2}+1+6+3\sqrt{2} < k+2$

$k < 7+5\sqrt{2} < k+2$

Hence, $k < 7+5\sqrt{2}$

$k < 14.07$

Now,

$7+5\sqrt{2} < k+2$

$k > 5+5\sqrt{2}$

$k > 12.07$

Hence, $k = 13$

15. (B) Let one root = α

then second root = $\alpha+2\sqrt{3}$

Given equation : $x^2 - kx + k = 0$

then sum of roots

$$\alpha + \alpha+2\sqrt{3} = \frac{-(-k)}{1}$$

$$2\alpha+2\sqrt{3} = k$$

$$\alpha+\sqrt{3} = \frac{k}{2}$$

$$\alpha = \frac{k}{2}-\sqrt{3} \quad ...(i)$$

and product of roots $\alpha(\alpha+2\sqrt{3}) = k$

$$\left(\frac{k}{2}-\sqrt{3}\right)\left(\frac{k}{2}-\sqrt{3}+2\sqrt{3}\right) = k$$

[from eqn. (i)]

$$\left(\frac{k}{2}-\sqrt{3}\right)\left(\frac{k}{2}+\sqrt{3}\right) = k$$

$$\frac{k^2}{4}-3 = k$$

$$k^2 - 12 = 4k$$

$$k^2 - 4k - 12 = 0$$

$$k^2 - 6k + 2k - 12 = 0$$

$$(k-6)(k+2) = 0$$

when, $k - 6 = 0$

then $k = 6$

when $k + 2 = 0$

then $k = -2$.

16. (A) By hit and trial method
Put $x = 5$ and $y = -5$
verifying the given equations

$$x+\frac{5}{y} = 5+\frac{5}{-5} = 4$$

and $y+\frac{5}{x} = -5+\frac{5}{5} = -4$

Hence, $x + y = 5 - 5 = 0$

17. (C) Given that $x^2 - x + 1 = 0$
Hence $x = -\omega_1, \omega^2$
$\therefore$ Put $x = -\omega$

$$\left(-\omega+\frac{1}{\omega}\right)^2+\left(-\omega+\frac{1}{\omega}\right)^4+\left(-\omega+\frac{1}{\omega}\right)^8$$

$(\omega^2 - \omega)^2 + (\omega^2 - \omega)^4 + (\omega^2 - \omega)^8$

$\left[\because \frac{1}{w} = w^2\right]$

$= (\omega^2-\omega)^2 [1 + (\omega^2-\omega)^2 \times (\omega^2-\omega)^6]$
$= (\omega^2 - \omega)^2 [1 + \omega^4 + \omega^2 - 2\omega^3 +$
$= (\omega^4 + \omega^2 - 2\omega^2)^3]$
$= (\omega^2 - \omega)^2 [1 + \omega + \omega^2 - 2 +$
$(\omega + \omega^2 - 2)^3]$
$= (\omega^2 - \omega)^2 [0 - 2 + (-1 - 2)^3]$
$= (\omega^2 - \omega)^2 [-2 - 27]$
$= -29 \times (\omega^4 + \omega^2 - 2\omega^3)$
$= -29 \times (\omega + \omega^2 - 2)$
$= -29 \times (-1 - 2)$
$= 87$

18. (C) If $x + y + z = k$

Then no. of positive integer solutions $= {}^{k-1}c_{r-1}$
$\therefore$ Required no. $= {}^{5-1}C_{3-1} = {}^4C_2 = 6$

19. (B) Given that
$S_k = 3k^2 + 5k$
$S_1 = 3 \times 1^2 + 5 \times 1 = 8$
$S_2 = 3 \times 2^2 + 5 \times 2 = 22$
$S_3 = 3 \times 3^2 + 5 \times 3 = 42$
$S_4 = 3 \times 4^2 + 5 \times 4 = 68$
Now $T_1 = S_2 - S_1 = 22 - 8 = 14$
$T_2 = S_3 - S_2 = 42 - 22 = 20$
and $T_3 = S_4 - S_3 = 68 - 42 = 26$
$\therefore$ Common difference $d = T_2 - T_1 = T_3 - T_2 = 6$
Hence, the terms of S form an arithmetic progression, with common difference 6.

20. (B) Let the first term of a G.P = a
and common ratio = r

$$S_8 = 5S_4$$

$$\frac{a(r^8-1)}{r-1} = \frac{5a(r^4-1)}{r-1}$$

$$(r^8 - 1) = 5(r^4 - 1)$$

$$(r^4 - 1)(r^4 + 1) = 5(r^4 - 1)$$

$$r^4 + 1 = 5$$

$$r^4 = 4$$

$$r^2 = \pm 2$$

$$r^2 = 2$$

[$\because r^2 \neq -2$, because r is a real number]

$\therefore \quad r = \pm\sqrt{2}$

21. (A) We know that if we subtract or add from each observation then mean is same

$\therefore$ Required arithmetic mean

$$= \frac{50-5}{20}$$

$$= \frac{45}{20}$$

$$= 2.25$$

22. (C) Arithmetic mean

$$= \frac{8^2+9^2+10^2+...+15^2}{8}$$

$$= \frac{(1^2+2^2+...+15^2)-(1^2+2^2+...+7^2)}{8}$$

$$= \frac{\frac{1}{6}\times 15\times(15+1)\times(2\times 15+1) - \frac{1}{6}\times 7\times(7+1)(2\times 7+1)}{8}$$

$$= \frac{\frac{1}{6}\times 15\times 16\times 31-\frac{1}{6}\times 7\times 8\times 15}{8}$$

$$= \frac{1}{6}\times 15\times 2\times 31-\frac{1}{6}\times 7\times 15$$

$$= \frac{15}{6}(62-7)$$

$$= \frac{15}{6}\times 55 \quad 5 \quad 55$$

$$= \frac{5\times 55}{2}$$

$$= 137.5$$

23. (B) ${}^{5n}C_r = {}^{5n}C_{n+r}$

$5n = r + n + r$

$4n = 2r$

$r = 2n$

24. (D) ${}^6C_3 + {}^6C_2 + {}^6C_1 + {}^6C_0$
$= 20 + 15 + 6 + 1$
$= 42$

25. (C) Required No : $= \frac{\lfloor 4 \lfloor 4}{\lfloor 2} = \frac{24\times 24}{2}$

$= 288$

26. (C) Given that diagonals = 20

$$\frac{n(n-3)}{2} = 20$$

$$n^2 - 3n = 40$$

$$n^2 - 3n - 40 = 0$$

$$n^2 - 8n + 5n - 40 = 0$$

$$n(n-8) + 5(n-8) = 0$$

$(n - 8)(n + 5) = 0$

Hence, $x = 8$

27. (C) Required No. $= \underline{|3}\ \underline{|2}$

$= 6 \times 2$

$= 12$

28. (A) Required probability $= \dfrac{{}^7C_3 \times {}^4C_3}{{}^{11}C_6}$

$$= \frac{\dfrac{\underline{|7}}{\underline{|4}\ \underline{|3}} \times \dfrac{\underline{|4}}{\underline{|3}\ \underline{|1}}}{\dfrac{\underline{|11}}{\underline{|6}\ \underline{|5}}}$$

$$= \frac{\dfrac{7\times6\times5}{3\times2\times1} \times \dfrac{4\times\underline{|3}}{\underline{|3}}}{\dfrac{11\times10\times9\times8\times7\underline{|6}}{\underline{|6}\times5\times4\times3\times2\times1}}$$

$$= \frac{35\times4}{11\times7\times3\times2} = \frac{10}{33}$$

29. (D) Required probability

$$= \frac{{}^4C_2 \times {}^7C_4 + {}^4C_3 \times {}^7C_3 + {}^4C_4 \times {}^7C_2}{{}^{11}C_6}$$

$$= \frac{\dfrac{\underline{|4}}{\underline{|2}\ \underline{|2}} \times \dfrac{\underline{|7}}{\underline{|4}\ \underline{|3}} \times \dfrac{\underline{|4}}{\underline{|3}} \dfrac{\underline{|7}}{\underline{|4}\underline{|3}} \times 1 \times \dfrac{\underline{|7}}{\underline{|5}\ \underline{|2}}}{\dfrac{\underline{|11}}{\underline{|6}\ \underline{|5}}}$$

$$= \frac{6\times35+4\times35+21}{\dfrac{11\times10\times9\times8\times7\times6}{5\times4\times3\times2\times1}}$$

$$= \frac{371}{66\times7} = \frac{53}{60}$$

30. (C) Let all variables = 1

∴ Sum of binomial coefficients

$(1 + 1)^n = 256$

$2^n = 256$

$2^n = 2^8$

$n = 8$

middle term $= \left(\dfrac{n}{2}+1\right)^{\text{th}}$ term

$= \left(\dfrac{8}{2}+1\right)^{\text{th}}$ term

$= 5^{\text{th}}$ term

Hence, 5^{th} term will be the greatest binomial coefficient.

31. (B) Given that mean $(\bar{x}) = 200$

$np = 200$ ...(i)

and variance $(\sigma^2) = 160$

$npq = 160$

$np(1 - p) = 160$ ...(ii)

on dividing eqn. (ii) by eqn. (i) we get.

$$\frac{np(1-p)}{np} = \frac{160}{200}$$

$1 - p = \dfrac{4}{5}$

$p = \dfrac{1}{5}$

put the value of p in eqn. (i)

$np = 200$

$n \times \dfrac{1}{5} = 200$

$n = 1000$

32. (C) Required No. of terms

$$= \frac{n}{\text{LCM of (2 and 4)}} + 1$$

$= \dfrac{12}{4} + 1$

$= 4$

33. (C) Let $\alpha = 30'$

$p = \tan 2\alpha - \tan \alpha$

$= \tan 60^\circ - \tan 30^\circ$

$= \sqrt{3} - \dfrac{1}{\sqrt{3}}$

and $q = \cot \alpha - \cot 2\alpha$

$= \cot 30 - \cot 60$

$= \sqrt{3} - \dfrac{1}{\sqrt{3}}$

∴ $p/q = 1$

from option (C) $\tan \alpha$, $\tan 2\alpha$ $= \tan 30^\circ \tan 60^\circ$

$= \dfrac{1}{\sqrt{3}} \times \sqrt{3}$

$= 1$

Hence, option (C) is correct.

34. (D) $p + q = \sqrt{3} - \dfrac{1}{\sqrt{3}} + \sqrt{3} - \dfrac{1}{\sqrt{3}}$

$= 2\sqrt{3} - \dfrac{2}{\sqrt{3}}$

$= \dfrac{4}{\sqrt{3}}$

from option (D)

$2 \operatorname{cosec} 4\alpha = 2 \operatorname{cosec} 120$

$= 2 \times \dfrac{2}{\sqrt{3}} = \dfrac{4}{\sqrt{3}}$

Hence, option (D) is correct.

35. (C) $\tan^2 \alpha = \tan^2 30 = \dfrac{1}{3}$

from option (C) $= \dfrac{p}{p + 2q}$

$$= \frac{\sqrt{3} - \dfrac{1}{\sqrt{3}}}{\sqrt{3} - \dfrac{1}{\sqrt{3}} + 2\sqrt{3} - \dfrac{2}{\sqrt{3}}}$$

$$= \frac{3-1}{3-1+6-2} = \frac{2}{6} = \frac{1}{3}$$

Hence, option (C) is correct.

36. (C) Given that

$2 \sin \alpha + \cos \alpha = 2$

On dividing $\cos \alpha$ both side, we get

$2 \tan \alpha + 1 = 2 \sec \alpha$

$\sec \alpha - \tan \alpha = \dfrac{1}{2}$ ..(i)

$\dfrac{1}{\sec \alpha - \tan \alpha} = 2$

$\sec \alpha + \tan \alpha = 2$ ...(ii)

On substracting eqn (i) from eqn (ii)

$2 \tan \alpha = 2 - \dfrac{1}{2}$

$\tan \alpha = \dfrac{3}{4}$

37. (B) $2 \sin 2\alpha + \cos 2\alpha =$

$$= \frac{2 \times \tan \alpha}{1 + \tan^2 \alpha} + \frac{1 - \tan^2 \alpha}{1 + \tan^2 \alpha}$$

$$= \frac{2 \times 2 \times \dfrac{3}{4}}{1 + \left(\dfrac{3}{4}\right)^2} + \frac{1 - \left(\dfrac{3}{4}\right)^2}{1 + \left(\dfrac{3}{4}\right)^2}$$

$$= \frac{3}{1 + \dfrac{9}{16}} + \frac{1 - \dfrac{9}{16}}{1 + \dfrac{9}{16}}$$

$$= \frac{3}{\dfrac{25}{16}} + \frac{\dfrac{7}{16}}{\dfrac{25}{16}}$$

$$= \frac{48}{25} + \frac{7}{25} = \frac{55}{25} = \frac{11}{5}$$

38. (A) $T_5 = a + 4d$

$T_7 = a + 6d$

$T_{13} = a + 12d$

Here a and d are the first term and common difference of an A.P.

Given that

$(T_7)^2 = T_5 \times T_{13}$

$(a + 6d)^2 = (a + 4d)(a + 12d)$

$a^2 + 36d^2 + 12ad = a^2 + 4ad + 12ad + 48d^2$

$12d^2 = -4ad$

$12d = -4a$

$\dfrac{a}{d} = \dfrac{12}{-4} = -3$

Hence, the ratio of first term of A.P. to its common difference is –3.

39. (C) Given that

∵ p, 1, q are in A.P.

then $p + q = 2$ ...(i)

and p_i 2, q and in G.P.

then $pq = 4$ ...(ii)

On dividing eqn. (i) by eqn. (ii), we get

$$\frac{p+q}{pq} = \frac{2}{4}$$

$$\frac{1}{p}+\frac{1}{q} = \frac{1}{2}$$

$$\frac{1}{p}+\frac{1}{q} = \frac{2}{4}$$

Hence, $\frac{1}{p}, \frac{1}{4}, \frac{1}{q}$ are in A.P.

and p, 4, q are also in H.P.

40. (B) $p + q + r = \sin 35^o + \sin 25^o + \sin(-95^o)$

$= 2 \sin 30^o \cos 5^o - \cos 5^o$

$= \cos 5^o - \cos 5^o = 0$

41. (A) $pq + qr + rp = \sin 35^o \sin 25^o + \sin 25^o \sin(-95^o) + \sin 35 \sin(-95^o)$

$= \sin 35^o \sin 25^o + \sin 25^o \cos 5^o - \sin 35^o \cos 5^o$

$= \frac{2}{2} \sin 35^o \sin 25^o - \cos 5^o (\sin 25^o + \sin 35^o)$

$= \frac{1}{2}[\cos 10^o - \cos 60^o]$

$- \cos 5^o [2 \sin 30^o \cos 5^o]$

$= \frac{1}{2}\cos 10 - \frac{1}{2}\times\frac{1}{2} - \cos^2 5^o$

$= \frac{1}{2}(2\cos^2 5^o - 1) - \frac{1}{4} - \cos^2 5^o$

$= \cos^2 5 - \frac{1}{2} - \frac{1}{4} - \cos^2 5^o$

$= -\frac{3}{4}$

42. (C) $(p+q+r)^2 = p^2 + q^2 + r^2 + 2(pq + qr + rp)$

$0 = p^2 + q^2 + r^2 + 2 \times -\frac{3}{4}$

$p^2 + q^2 + r^2 = \frac{3}{2}$

43. (A) $p = |\sin \alpha + \cos \alpha|$

modules never gives negative value

Hence, minimum value = 0

44. (B) Maximum value = $\sqrt{a^2+b^2}$

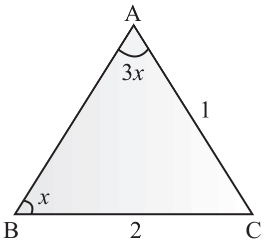
[$\because$ max value of $a \sin \theta + b \cos \theta$

$= \sqrt{a^2+b^2}$]

$= \sqrt{1^2+1^2}$

$= \sqrt{2}$

45. (B)



Using Sine rule

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\frac{\sin 3x}{2y} = \frac{\sin x}{y}$$

$3 \sin x - 4 \sin^3 x = 2 \sin x$

$3 - 4 \sin^2 x = 2$

$4 \sin^2 x = 1$

$\sin x = \frac{1}{2}$

$x = 30^o$

Hence, option (B) is correct.

46. (C) **I.** Given that angles are in the ratio of 3 : 1

then second angle = 3 × 30º = 90º

Hence, given triangle is right angled triangle.

II. Statement-II is incorrect because one side of a triangle is 2 times the other.

III. 1st angle = 30º

2nd angle = 90º

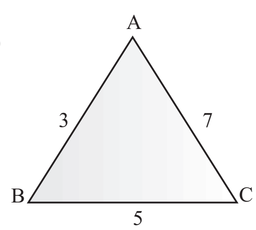
3rd angle = 180° – 30º – 90º = 60º

Hence, angles all in AP because

2 × 3rd angle = 1st angle + 2nd angle.

Hence, statements I and III only are correct.

47. (C)



$$\cos B = \frac{3^2+5^2-7^2}{2\times3\times5} = \frac{9+25-49}{30}$$

$\cos B = \frac{-1}{2}$

$\cos B = \cos 120^o$

$B = 120^o$

Now $\angle A + \angle C = 180 - 120 = 60$

So sum of acute angles ($\angle$A and $\angle$C) of the triangle is also acute.

Hence, both statements are correct.

48. (C) $\angle B = 120^o$

49. (A) Area = $\frac{1}{2}ac \sin B$

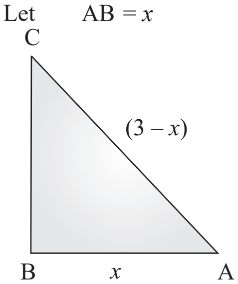
$= \frac{1}{2}\times 3\times 5 \sin 120^o$

$= \frac{1}{2}\times 15\times\frac{\sqrt{3}}{2}$

$= \frac{15\sqrt{3}}{4}\text{cm}^2$

50. (C) Given that AB + AC = 3 units

Let AB = x



then AC = $3 - x$

$\therefore$ $BC^2 = (3-x)^2 - x^2$

$= 9 + x^2 - 6x - x^2$

$= 9 - 6x$

$\therefore$ BC = $\sqrt{9-6x}$ units

Now,

Area = $\frac{1}{2}$ AB × BC

$A = \frac{1}{2}\times x\times\sqrt{9-6x}$

On squaring both sides, we get

$A^2 = \frac{1}{4}x^2(9-6x) = \frac{1}{4}(9x^2-6x^3)$

at $A^2 = p$

On differentiating both sides, we get

$\frac{dp}{dx} = \frac{1}{4}(18x - 18x^2)$

put $\frac{dp}{dx} = 0$

then $\frac{1}{4}(18x - 18x^2) = 0$

$18x = 18x^2$

$x = 1$

$\frac{dp^2}{dx^2} = \frac{1}{4}(18 - 36x)$

$\left.\frac{dp^2}{dx^2}\right|_{x=1} = \frac{1}{4}(18-36) < 0$

Hence, p is max. at $x = 1$

Now A^2 will be max. at $x = 1$

$\therefore$ AB = 1

AC = 3 – 1 = 2

BC = $\sqrt{9-6x} = \sqrt{9-6}$

$= \sqrt{3}$

In ΔABC;

$\tan A = \frac{BC}{AB} = \frac{\sqrt{3}}{1}$

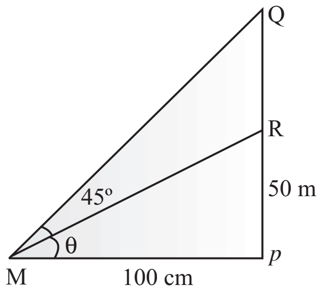
$\tan A = \tan 60^o$

$\therefore \qquad A = 60^\circ \text{ or } \frac{\pi}{3}$

51. (A) $A = \frac{1}{2} \times AB \times BC$

$= \frac{1}{2} \times 1 \times \sqrt{3} = \frac{\sqrt{3}}{2}$ = square unit

52. (B)



In Δ RMP

$\frac{RP}{MP} = \tan\theta$

$\frac{50}{100} = \tan\theta$

$\tan\theta = \frac{1}{2}$

$\theta = \tan^{-1}\frac{1}{2}$

Now $\angle RMQ = 45^\circ - \tan^{-1}\frac{1}{2}$

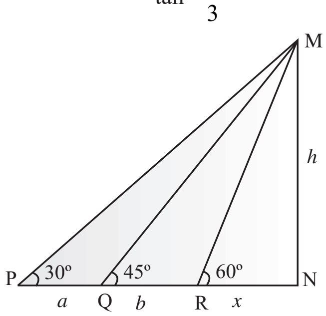
$= \tan^{-1} 1 - \tan^{-1}\frac{1}{2}$

$= \tan^{-1}\left(\frac{1-\frac{1}{2}}{1+1\times\frac{1}{2}}\right)$

$\angle RMQ = \tan^{-1}\frac{\frac{1}{2}}{\frac{3}{2}}$

$= \tan^{-1}\frac{1}{3}$

53. (B)



In ΔMQN

$\frac{h}{b+x} = \tan 45^\circ$

$h = b + x \qquad$ ...(i)

In ΔMRN

$\frac{h}{x} = \tan 60^\circ$

$x = \frac{h}{\sqrt{3}} \qquad$ ...(ii)

from eq^n (i)

$h = b + \frac{h}{\sqrt{3}}$

$\Rightarrow \qquad b = h - \frac{h}{\sqrt{3}}$

$= \frac{\sqrt{3}h - h}{\sqrt{3}}$

$h = \frac{b\sqrt{3}}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$

$= \frac{b\sqrt{3}(\sqrt{3}+1)}{3-1}$

$= \frac{b\sqrt{3}(\sqrt{3}+1)}{2}$

$= \left(\frac{3+\sqrt{3}}{2}\right)b \qquad$ ...(iii)

Hence $\quad PN = a + b + x$

$= a + \frac{2h}{3+\sqrt{3}} + \frac{h}{\sqrt{3}}$

[from eqn (ii) and (iii)]

$= a + \frac{2h}{3+\sqrt{3}} \times \frac{3-\sqrt{3}}{3-\sqrt{3}} + \frac{h\sqrt{3}}{3}$

$= a + \frac{6h - 2h\sqrt{3}}{9-3} + \frac{h\sqrt{3}}{3}$

$= a + h - \frac{h\sqrt{3}}{3} + \frac{h\sqrt{3}}{3}$

$= a + h$

In ΔMPN

$\frac{h}{PN} = \tan 30^\circ$

$\frac{h}{a+h} = \frac{1}{\sqrt{3}}$

$a + h = h\sqrt{3}$

$h = \frac{a}{\sqrt{3}-1}$

$= \frac{a(\sqrt{3}+1)}{2}$

Hence PN = $a + h$

$= a + a\left(\frac{\sqrt{3}+1}{2}\right)$

$= a\left(\frac{2+\sqrt{3}+1}{2}\right)$

$= \left(\frac{3+\sqrt{3}}{2}\right)a$

54. (A) $MN = \left(\frac{3+\sqrt{3}}{2}\right)b$

55. (D) Given that k is root of $x^2 - 4x + 1 = 0$
then $k^2 - 4k + 1 = 0$

$k = \frac{+4 \pm \sqrt{16 - 4\times1\times1}}{2\times1}$

$= \frac{4 \pm \sqrt{12}}{2}$

$= \frac{4 \pm 2\sqrt{3}}{2}$

$= 2 \pm \sqrt{3}$

$\therefore \tan^{-1} k + \tan^{-1}\frac{1}{k}$

$= \tan^{-1}(2+\sqrt{3}) + \tan^{-1}(2-\sqrt{3})$

$= \tan^{-1}\frac{2+\sqrt{3}+2-\sqrt{3}}{1-(2+\sqrt{3})(2-\sqrt{3})}$

$= \tan^{-1}\frac{4}{1-4+3}$

$= \tan^{-1}\frac{4}{0}$

$= \tan^{-1}\infty$

$= \frac{\pi}{2}$

56. (C) $\tan^{-1} k + \tan^{-1}\frac{1}{2} = \frac{\pi}{4}$

$\tan^{-1}k = \tan^{-1} 1 - \tan^{-1}\frac{1}{2}$

$= \tan^{-1}\frac{1-\frac{1}{2}}{1+\frac{1}{2}}$

$\tan^{-1} k = \tan^{-1}\frac{1}{3} \Rightarrow k = \frac{1}{3}$

57. (D) Given that

$y = \sin^{-1}\left(x - \frac{4x^3}{27}\right)$

$y = \sin^{-1}\left[3\times\frac{x}{3} - 4\left(\frac{x}{3}\right)^3\right]$

$y = 3\sin^{-1}\left(\frac{x}{3}\right)$

$[\because 3\sin^{-1}\theta = \sin^{-1}(3\theta - 4\theta^3)]$

58. (C) $y = 3\sin^{-1}\left(\frac{x}{3}\right)$

differentiating with respect to x weget

$\frac{dy}{dx} = \frac{3}{\sqrt{1-\left(\frac{x}{3}\right)^2}} \times \frac{1}{3}$

$= \frac{3}{\sqrt{9-x^2}}$

59. (D) $[x \;\; 1 \;\; 1]\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\\7 & 8 & 9\end{bmatrix}\begin{bmatrix}1\\1\\x\end{bmatrix} = [45]$

$$[x + 11 \quad 2x + 13 \quad 3x + 15]\begin{bmatrix}1\\1\\x\end{bmatrix}$$

$= [45]$

$[x + 11 + 2x + 13 + 3x^2 + 15x]$

$= [45]$

$3x^2 + 18x + 24 = 45$

$3x^2 + 18x - 21 = 0$

$x^2 + 6x - 7 = 0$

$$x = \frac{-6 \pm \sqrt{36 - 4\times1\times-7}}{2\times1}$$

$$= \frac{-6\pm\sqrt{36+28}}{2} = \frac{-6\pm 8}{2}$$

$= 1, -7.$

Hence $x = 1$

60. (B) Given that $A = \begin{bmatrix}y & z & x\\ z & x & y\\ x & y & z\end{bmatrix}$

$$A' = \begin{bmatrix}y & z & x\\ z & x & y\\ x & y & z\end{bmatrix}$$

$AA' =$

$$\begin{bmatrix}x^2+y^2+z^2 & yz+zx+xy & xy+zy+xz\\ xy+yz+zx & x^2+y^2+z^2 & xy+zy+xz\\ xy+yz+zx & xy+yz+zx & x^2+y^2+z^2\end{bmatrix}$$

$$= \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

Hence $x^2 + y^2 + z^2 = 1$.

61. (D) $f(\theta) = \begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$

$$f(\pi) = \begin{bmatrix}\cos\pi & \sin\pi\\ -\sin\pi & \cos\pi\end{bmatrix}$$

$$= \begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix}$$

$$[f(\pi)]^2 = \begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix}\begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix}$$

$$= \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

62. (D) $A = \begin{bmatrix}1 & 2 & 2\\ 2 & 1 & 2\\ 2 & 2 & 1\end{bmatrix}$

$$A^2 = \begin{bmatrix}1 & 2 & 2\\ 2 & 1 & 2\\ 2 & 2 & 1\end{bmatrix}\begin{bmatrix}1 & 2 & 2\\ 2 & 1 & 2\\ 2 & 2 & 1\end{bmatrix}$$

$$= \begin{bmatrix}9 & 8 & 8\\ 8 & 9 & 8\\ 8 & 8 & 9\end{bmatrix}$$

$$A^2 - 4A = \begin{bmatrix}9 & 8 & 8\\ 8 & 9 & 8\\ 8 & 8 & 9\end{bmatrix} - 4\begin{bmatrix}1 & 2 & 2\\ 2 & 1 & 2\\ 2 & 2 & 1\end{bmatrix}$$

$$= \begin{bmatrix}9 & 8 & 8\\ 8 & 9 & 8\\ 8 & 8 & 9\end{bmatrix} - \begin{bmatrix}4 & 8 & 8\\ 8 & 4 & 8\\ 8 & 8 & 4\end{bmatrix}$$

$$= \begin{bmatrix}5 & 0 & 0\\ 0 & 5 & 0\\ 0 & 0 & 5\end{bmatrix}$$

$$= 5\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix} = 5\,I_3$$

63. (B) If A is orthogonal matrix then $A^2 = I$

64. (C) I. Determinant of a square of matrix is equal to square determinant of matrices

II. $|M^{-1}| = \frac{1}{|M|}$

Hence $|M| \neq |M^{-1}|$

III. Determinant of a matrix is equal to determinant of a transposed matrix.

Hence, Statement I and III are correct. So two statements are correct.

65. (A) Given that $D = \begin{vmatrix}a & b & c\\ d & e & f\\ g & h & i\end{vmatrix}$

$$\therefore bB + cC - dD - gG = -b\begin{vmatrix}d & f\\ g & i\end{vmatrix}$$

$$+c\begin{vmatrix}d & e\\ g & h\end{vmatrix} + d\begin{vmatrix}b & c\\ h & i\end{vmatrix} + g\begin{vmatrix}b & c\\ e & f\end{vmatrix}$$

$= -b(di - fg) + c(dh - eg) + d(bi - ch) - g(bf - ce)$

$= -bdi + bfg + cdh - ceg + dbi - cdh - g\,bf + ceg$

$= 0$

66. (C) $\Delta = \begin{vmatrix}k(k+2) & 2k+1 & 1\\ 2k+1 & k+2 & 1\\ 3 & 3 & 1\end{vmatrix}$

$= (k^2 + 2k)(k + 2 - 3) - (2k + 1)(2k + 1 - 3) + 1(6k + 3 - 3k - 6)$

$= (k^2 + 2k)(k - 1) - (2k + 1)(2k - 2) + (3k - 3)$

$= k^3 + 2k^2 - k^2 - 2k - 4k^2 - 2k + 4k + 2 + 3k - 3$

$= k^3 - 3k^2 + 3k - 1$

$= (k - 1)^3$

I. If $k > 0$ then Δ is positive

II. If $k < 0$ then Δ is negative

III. If $k = 0$ then $\Delta \neq 0$

Hence, option (C) is correct.

67. (B) $\begin{vmatrix}2 & 3+i & -1\\ 3-i & 0 & i-1\\ -1 & -1-i & 1\end{vmatrix} = A + iB$

$2[0 - (i - 1)(-1 - i)] - (3 + i)[(3 - i) + i - 1] - 1[(3 - i)(-1 - i) - 0] = A + iB$

$-2(-i + 1 - i^2 + i) - (3 + i) \times 2[-3 + i - 3i + i^2] = A + iB$

$-4 - 6 - 2i - (-3 + i - 3i - 1) = A + iB$

$-10 - 2i - (-4 - 2i) = A + iB$

$-10 - 2i + 4 + 2i = A + iB$

$A + iB = -6$

68. (B) Given the $A^2 + B^2 + C^2 = 0$

Let $A = B = C = 0$

then $\begin{vmatrix}1 & \cos C & \cos B\\ \cos C & 1 & \cos A\\ \cos B & \cos A & 1\end{vmatrix}$

$$= \begin{vmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{vmatrix} = 0$$

69. (A) Given that

$$\begin{vmatrix}x+1 & \omega & \omega^2\\ \omega & x+\omega^2 & 1\\ \omega^2 & 1 & x+\omega\end{vmatrix} = 0$$

$C_1 \to C_1 + C_2 + C_3$

$$\begin{vmatrix}x+1+\omega+\omega^2 & \omega & \omega^2\\ x+1+\omega+\omega^2 & x+\omega^2 & 1\\ x+1+\omega+\omega^2 & 1 & x+\omega\end{vmatrix} = 0$$

$$(x+1+\omega+\omega^2)\begin{vmatrix}1 & \omega & \omega^2\\ 1 & x+\omega^2 & 1\\ 1 & 1 & x+\omega\end{vmatrix}$$

$= 0$

$\because x + 1 + \omega + \omega^2 = 0$

$x = 0\ [\because 1 + \omega + \omega^2 = 0]$

70. (A) Slope if 1st line $= \dfrac{-\text{coefficient of } x}{\text{coefficient of } y}$

$$= \frac{-m^2}{n}$$

Slope of 2nd line = $\frac{n^2}{m}$

For perpendicular,

$$\frac{-m^2}{n} \times \frac{n^2}{m} = -1$$

$$-mn = -1$$

$$mn - 1 = 0$$

71. (D) Let

A = (p, 1)
B = (1, 2)
C = (0, 0)

$\therefore$ $AB = \sqrt{(1-p)^2 + (q-1)^2}$

$BC = \sqrt{(0-1)^2 + (0-q)^2}$

$= \sqrt{q^2 + 1}$

$AC = \sqrt{(0-p)^2 + (0-1)^2}$

$= \sqrt{p^2 + 1}$

For equilateral triangle

BC = AC

$\sqrt{q^2+1} = \sqrt{p^2+1}$

on squaring both sides, we get

$p^2 + 1 = q^2 + 1$

$p = q$

and AB = BC

$\sqrt{(1-p)^2 + (1-q)^2} = \sqrt{q^2+1}$

$1 + p^2 - 2p + q^2 + 1 - 2q = q^2 + 1$

$1 + q^2 - 2q - 2q = 0$

$q^2 - 4q + 1 = 0$

$$q = \frac{+4 \pm \sqrt{16 - 4\times1\times1}}{2\times1}$$

$$= \frac{4 \pm \sqrt{12}}{2}$$

$$= \frac{4 \pm 2\sqrt{3}}{2}$$

$$= 2 \pm \sqrt{3}$$

$\therefore$ $p = q = 2 \pm \sqrt{3}$

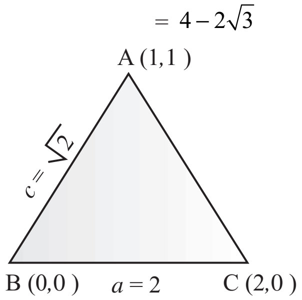
Now $p + q = (2 + \sqrt{3} + 2 + \sqrt{3})$

$= 4 + 2\sqrt{3}$

or $p + q = 2 - \sqrt{3} + 2 - \sqrt{3}$

$= 4 - 2\sqrt{3}$

72. (A)



$AB = \sqrt{(1-0)^2 + (1-0)^2} = \sqrt{2}$

$BC = \sqrt{(2-0)^2 + 0} = 2$

$AC = \sqrt{(2-1)^2 + (0-1)^2} = \sqrt{2}$

Coordinate of P =

$$\left(\frac{ax_1 + bx_2 + cx_3}{a+b+c}, \frac{ay_1 + by_2 + cy_3}{a+b+c}\right)$$

$$= \left(\frac{2\times1 + \sqrt{2}\times0 + \sqrt{2}\times2}{\sqrt{2}+\sqrt{2}+2}, \frac{2\times1 + \sqrt{2}\times0 + \sqrt{2}\times0}{\sqrt{2}+\sqrt{2}+2}\right)$$

$$= \left(\frac{2+2\sqrt{2}}{2+2\sqrt{2}}, \frac{2}{2+2\sqrt{2}}\right)$$

$$= \left(1, \frac{1}{1+\sqrt{2}}\right)$$

$$= (1, \sqrt{2} - 1)$$

73. (B) Coordinate of P = $\left(\frac{3+1}{2}, \frac{-1+1}{2}\right)$

= (2, 0)

Let the Coordinate of Q = (x, y)

Distance between P and Q =

$\sqrt{(2-x)^2 + (0-y)^2} = \sqrt{2}$

$(2-x)^2 + y^2 = 2$

$4 + x^2 - 4x + y^2 = 2$

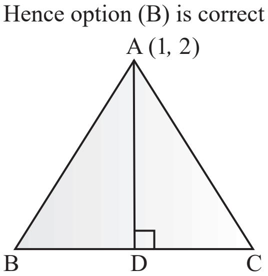
$x^2 - 4x + y^2 + 2 = 0$

from option (B)

$3^2 - 4 \times 3 + 1^2 + 2 = 0$

Hence option (B) is correct

74. (D)



B(–2, 6)

Slope of AD (m_1) = $\frac{6-2}{-2-1} = \frac{-4}{3}$

Slope of BC = $-\frac{1}{m_1} = \frac{3}{4}$

Equation of BC

$y - y_1 = m(x - x_1)$

$y - 6 = \frac{3}{4}(x + 2)$

$4y - 24 = 3x + 6$

$3x - 4y + 6 + 24 = 0$

$3x - 4y + 30 = 0$

75. (B) Given that Diameter = 10 cm

$\therefore$ radius = 5 cm

from option (B), circle has a radius of 5 units.

Hence, option (B) is correct.

76. (C) Given circle

$x^2 + y^2 + 2x + 2y + 1 = 0$

On comparing with

$x^2 + y^2 + 2gx + 2fy + c = 0$

$g = +1$

$f = +1$

$c = 1$

$\therefore$ centre = (– 1, – 1)

and radius = $\sqrt{g^2 + f^2 - c}$

$= \sqrt{1^2 + 1^2 - 1} = 1$

General form of circle

$= (x_1 \pm r\cos\theta, y_1 \pm r\sin\theta)$

$= (-1 \pm 1 \times \cos 45, -1 \pm 1 \times \cos 45)$

$= \left(-1 + \frac{1}{\sqrt{2}}, -1 - \frac{1}{\sqrt{2}}\right)$

[Signs are taken according to quadrant]

77. (A) Given that

$\frac{dy}{dx} = 4$

$dy = 4dx$

On integrating both sides, we get

$\int dy = \int 4dx$

$y = 4x$

Hence, it is a straight lines passing through (1, 4)

78. (D) $\theta = 45°$

$\therefore$ $m = \pm \tan\theta$

$m = \tan 45° = 1$

Given that $y^2 = 4x$

Here, $a = 1$

$\therefore$ Required Point = $\left(\frac{a}{m^2}, \frac{2a}{m}\right)$

$= \left(\frac{1}{1}, \frac{2}{1}\right) = (1, 2)$

79. (B) $25x^2 - 75y^2 = 225$

$\frac{x^2}{9} - \frac{y^2}{3} = 1$

$\frac{x^2}{3^2} - \frac{y^2}{(\sqrt{3})^2} = 1$

Required distance = $2c$

$= 2\sqrt{a^2 + b^2}$

$= 2\sqrt{9+3}$

$= 2\sqrt{12}$

$= 4\sqrt{3}$ units

80. (A) $c = \sqrt{b^2 - a^2} = \sqrt{25-9}$

$c = 4$

Required eccentricity $= \frac{c}{a} = \frac{4}{3}$

81. (C) $\lim_{x\to 0} \frac{\sqrt{f(x)}-3}{\sqrt{f(x)+7}-4}$

$= \lim_{x\to 0} \frac{\sqrt{x^2+9}-3}{\sqrt{x^2+9+7}-4}$

$= \lim_{x\to 0} \frac{\sqrt{x^2+9}-3}{\sqrt{x^2+16}-4}$

$= \lim_{x\to 0} \frac{\sqrt{x^2+9}-3}{\sqrt{x^2+16}-4} \times \frac{\sqrt{x^2+9}+3}{\sqrt{x^2+9}+3} \times \frac{\sqrt{x^2+16}+4}{\sqrt{x^2+16}+4}$

(on rationalizing)

$= \lim_{x\to 0} \frac{x^2+9-9}{x^2+16-16} \times \frac{\sqrt{x^2+16}+4}{\sqrt{x^2+9}+3}$

$= \lim_{x\to 0} \frac{\sqrt{x^2+16}+4}{\sqrt{x^2+9}+3}$

$= \frac{\sqrt{16}+4}{\sqrt{9}+3} = \frac{4+4}{3+3} = \frac{8}{6} = \frac{4}{3}$

82. (B) Given that $f(x) = x^2 + 9$

$f'(x) = 2x$

Statement I : It is not correct because it is possible only when $x > 0$ but it is not given.

It can be increased or decreased both.

Statement II : $f'(x) = 2x$

$f''(x) = 2 > 0$

Here $f(x)$ is maximum

So, statement II is correct.

83. (B) $f(x) = \sin [x]$

$= \sin [0] = 0$

$g(x) = [x] = [0] = 0$

Hence, $\lim_{x\to 0} \{f(x)g(x)\} = \lim_{x\to 0} (0 \times 0) = 0$

84. (D) $\lim_{x+0} \frac{f(x)}{g(x)} = \lim_{x+0} \frac{0}{0}$

Hence, limit does not exist.

85. (A) Given that $f(x) = x^2 - 1$

$\therefore fof(x) = f(x^2 - 1) = (x^2 - 1)^2 - 1$

$= x^4 + 1 - 2x^2 - 1$

$= x^4 - 2x^2$

$\therefore \lim_{x\to 1}\{fof(x)\} = \lim_{x\to 1}(x^4 - 2x^2)$

$= 1 - 2$

$= -1$

86.(C) Given that

$$f(x) = \begin{cases} x^3, & x^2 < 1 \\ x^2, & x^2 > 1 \end{cases}$$

$\therefore \lim_{x\to 0} f'(x) = \lim_{x\to 0}\left(\frac{d}{dx}x^3\right) = \lim_{x\to 0} 3x^2$

$= 3\lim_{x\to 0} x^2 = 0$

87. (D) Statement I.

LHL $= \lim_{x\to -1^-} f(x-h)$

$= \lim_{h\to 0} f(-1-h)$

$= \lim_{h\to 0} (-1-h)^3$

$= -1$

RHL $= \lim_{h\to -1^+} f(x+h)$

$= \lim_{h\to 0} f(-1+h)$

$= \lim_{h\to 0} f(-1+h)^2 = 1$

LHL $\neq$ RHL

then the function is not continuous at $x = -1$

Statement II.

LHD $= \lim_{x\to 1^-} \frac{f(x)-f(1)}{x-1}$

$= \lim_{h\to 0} \frac{f(1-h)-f(1)}{1-h-1}$

$= \lim_{h\to 0} \frac{(1-h)^3-1}{-h}$

$= \lim_{h\to 0} \frac{1-h^3-3h+3h^2-1}{-h}$

$= \lim_{h\to 0} \frac{-h(h^2+3-3h)}{-h}$

$= 3$

RHD $= \lim_{h\to 1^+} \frac{f(x)-f(1)}{x-1}$

$= \lim_{h\to 0} \frac{f(1+h)-1}{1+h-1}$

$= \lim_{h\to 0} \frac{(1+h)^2-1}{h}$

$= \lim_{h\to 0} \frac{1+h^2+2h-1}{h}$

$= \lim_{h\to 0} (h+2) = 2$

LHD $\neq$ RHD

The function is not differentiable at $x = 1$

Hence, neither statement I nor II is correct.

88. (D) $(x + y)^{p+q} = x^p y^q$

On taking log both sides, we get

$(p + q) \log(x + y) = \log x^p + \log y^q$

$(p + q) \log(x + y) = p \log x + q \log y$

On differentiating both sides, we get

$(p+q)\frac{1}{x+y}\left(1+\frac{dy}{dx}\right) = \frac{p}{x} + \frac{q}{y}\frac{dy}{dx}$

$\frac{(p+q)}{x+y} + \frac{p+q}{x+y}\frac{dy}{dx} = \frac{p}{x} + \frac{q}{y}\frac{dy}{dx}$

$\frac{dy}{dx}\left[\frac{p+q}{x+y} - \frac{q}{y}\right] = \frac{p}{x} - \frac{p+q}{x+y}$

$\frac{dy}{dx}\left[\frac{py+qy-qx-qy}{y(x+y)}\right]$

$= \frac{px+py-px-qx}{x(x+y)}$

$\frac{dy}{dx}\left(\frac{py-qx}{y}\right) = \frac{(py-qx)}{x}$

$\frac{dy}{dx} = \frac{y}{x}$

Hence, $\frac{dy}{dx}$ is independent of both p and q.

89. (A) Since $\frac{dy}{dx}$ is independent of both p and q

then $\frac{dy}{dx}$ will always be $\frac{y}{x}$

90. (B) $\frac{1}{1-\cos x} = \frac{1}{1-1+2\sin^2\frac{x}{2}}$

$= \frac{1}{2}\text{cosce}^2\frac{x}{2}$

$\int y\,dx = \int \frac{1}{2}\text{cosce}^2\frac{x}{2}dx$

$= \frac{1}{2} \times \int \text{cosec}^2\frac{x}{2}dx$

$= \frac{1}{2} \times -2\cot\frac{x}{2} + c$

$= -\cot\frac{x}{2} + c$

91. (C) Required Area $= \int_0^4 4x\,dx$

$= 4\left[\frac{x^2}{2}\right]_0^4$

$= 4 \times \frac{16}{2}$

$= 32$ square units

92. (D) $y = 3$

$x - 3 = 3$

$x = 6$

Hence, Required Area

$= \int_0^3 -(x-3)dx + \int_3^6 (x-3)dx$

$= \left[-\frac{x^2}{2}+3x\right]_0^3 + \left[\frac{x^2}{2}-3x\right]_3^6$

$= \left[\frac{-9}{2}+9\right]+\left[18-18-\frac{9}{2}+9\right]$

$= \frac{9}{2}+\frac{9}{2}$

= 9 square units

93. (C) Given that $x^2 - 1 = f(x)$

on x-axis, $f(x) = 0$

$x^2 - 1 = 0$

$x = \pm 1$

Hence, Required Area

$= \left|\int_{-1}^0 (x^2-1)dx + \int_0^1 (x^2-1)dx\right|$

$= \left|\left[\frac{x^3}{3}-x\right]_{-1}^0 + \left[\frac{x^3}{3}-x\right]_0^1\right|$

$= \left|\left[0-\frac{(-1)^3}{3}-1\right]+\left[\frac{1}{3}-1-0\right]\right|$

$= \left|\frac{1}{3}-1+\frac{1}{3}-1\right|$

$= \left|\frac{2}{3}-2\right| = \frac{4}{3}$ square unit

94. (C) Given that

$x = (\text{sce}\ \theta - \cos\theta)$ and $y = \text{sce}^4\theta - \cos^4\theta$ on differentiating w.r.t θ

$\frac{dx}{d\theta} = \sec\theta\tan\theta + \sin\theta$

$\frac{dy}{d\theta} = 4\ \text{sce}^3\theta \times \sec\theta\tan\theta - 4\cos^3\theta \times -\sin\theta$

$= 4\sec^4\theta\tan\theta + 4\sin\theta\cos^3\theta$

Now,

$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$

$= \frac{4\sec^4\theta\tan\theta + 4\sin\theta\cos^3\theta}{\sec\theta\tan\theta+\sin\theta}$

$= \frac{4\tan\theta(\sec^4\theta+\cos^4\theta)}{\tan\theta(\sec\theta+\cos\theta)}$

$= \frac{4\sqrt{(\sec^4\theta-\cos^4\theta)^2+4\sec^4\theta\cos^4\theta}}{\sqrt{(\sec\theta-\cos\theta)^2+4\sec\theta\cos\theta}}$

$\frac{dy}{dx} = \frac{4\sqrt{y^2+4}}{\sqrt{x^2+4}}$

$\left(\frac{dy}{dx}\right)^2 = \frac{16(y^2+4)}{x^2+4}$

95. (C) $\left(\frac{dy}{dx}\right)^2 = \frac{16(y^2+4)}{x^2+4}$

On differentiating both sides

$\frac{2.dy}{dx}\frac{d^2y}{dx^2}$

$= \frac{16\left[(x^2+4)\,2y\frac{dy}{dx}-(y^2+4)2x\right]}{(x^2+4)^2}$

$(x^2+4)^2\frac{dy}{dx}\frac{d^2y}{dx}$

$= 16\left[(x^2+4)\frac{dy}{dx}-(y^2+4)x\right]$

On dividing by (y^2+4) on both sides we get

$\frac{(x^2+4)^2}{y^2+4}\frac{dy}{dx}\frac{d^2y}{dx}$

$= 16\left[\frac{(x^2+4)y\frac{dy}{dx}}{y^2+4}+x\right]$

$\frac{(x^2+4)^2}{y^2+4}\frac{dy}{dx}\cdot\frac{d^2y}{dx^2}$

$-\frac{16(y^2+4)}{x^2+4} = -16x$

$\left(\frac{x^2+4}{y^2+4}\right)\frac{dy}{dx}\left[(x^2+4)\frac{d^2y}{dx^2}-16y\right] =$

$-16x$

96. (D) $\vec{a}.\vec{b}$

$= (\sin^2\alpha)\hat{i}+(\sin^2\beta)\hat{j}+(\sin^2\gamma)\hat{j}$

$= \sin^2\alpha + \sin^2\beta + \sin^2\gamma$

$= 1 - \cos^2\alpha + 1 - \cos^2\beta + 1 - \cos^2\gamma$

$= 3 - (\cos^2\alpha + \cos^2\beta + \cos^2\gamma)$

$= 3 - 1 = 2$

97. (C) Both statements are correct because both statements are the properties of scalar triple product.

98. (B) Given that

$3\vec{a} - 4\vec{b} + \vec{c} = 0$

$\vec{c} = 4\vec{b} - 3\vec{a}$

$\overrightarrow{AB} = \vec{b} - \vec{a}$

$\overrightarrow{BC} = \vec{c} - \vec{b}$

$= 4\vec{b} - 3\vec{a} - \vec{b}$

$= 3\vec{b} - 3\vec{a}$

$\overrightarrow{BC} = 3(\vec{b} - \vec{a})$

$\overrightarrow{BC} = 3\overrightarrow{AB}$

$\frac{\overrightarrow{AB}}{\overrightarrow{BC}} = \frac{1}{3}$

$\overrightarrow{AB} : \overrightarrow{BC} = 1 : 3$

99. (A) Let $\theta = 45^o$

$\therefore\ \vec{c} = (\cos^2\theta)\ \vec{a} + (\sin^2\theta)\ \vec{b}$

$\vec{c} = \frac{\vec{a}}{2}+\frac{\vec{b}}{2}$

Now, $\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a} =$

$(\hat{i}\times\hat{j})+(\hat{j}\times\hat{k})+(\hat{k}\times\hat{i})$

$= \hat{k}-\frac{\hat{k}}{2}-\frac{\hat{k}}{2} = \vec{0}$

100. (A) $|\vec{a}\times\vec{b}| = |\vec{a}||\vec{b}|.\sin\theta$

$1 = \sin\theta$

[$\vec{a}\times\vec{b}$ are unit vectors]

$\theta = \frac{\pi}{2}$

$\therefore\ \vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

$= 1\times1\times\cos\frac{\pi}{2} = 0$

101. (B) We know that

$\cos(\alpha+\beta)\cos(\alpha-\beta) = \cos^2\alpha - \sin^2\beta$

$= \cos^2\alpha - (1 - \cos^2\beta)$

$= \cos^2\alpha - 1 + \cos^2\beta$

$= -1 + 1 - \cos^2\gamma$

$[\because \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1]$

$= -\cos^2\gamma$

102. (C)

$AB = \sqrt{(2-1)^2+(5-2)^2+(-2+1)^2}$

$= \sqrt{1+9+1} = \sqrt{11}$

$BC = \sqrt{(4-2)^2+(4-5)^2+(-3+2)^2}$

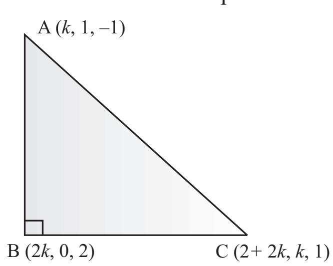
$= \sqrt{4+1+1} = \sqrt{6}$

Required area = AB × BC

$= \sqrt{11}\times\sqrt{6}$

$= \sqrt{66}$ square units

103. (D)



Direction ratios of AB = $(2k - k, 0 - 1, + 2 + 1)$

$= (k, -1, 3)$

Direction ratios of BC = $(2 + 2k - 2k, k - 0, 1 - 2)$

$= (2, k, -1)$

If B is right angle then product of direction ratios will be = 0.

$2k - k - 3 = 0$

$k = 3$

104. (A) Given that

$p = 2q = 3r$

$\frac{p}{6} = \frac{q}{3} = \frac{r}{2} = k$

then $p = 6k$, $q = 3k$, $r = 2k$

$\therefore \frac{x+1}{6k} = \frac{y-1}{3k} = \frac{z-2}{2k}$

$\frac{x+1}{6} = \frac{y-1}{3} = \frac{z-2}{2}$

$\therefore \cos\theta = \frac{3}{\sqrt{6^2+3^2+2^2}}$

[$\because$ The line makes an angle with y axis]

$\cos\theta = \frac{3}{7}$

Now $\cos 2\theta = 2\cos^2\theta - 1$

$= 2\times\left(\frac{3}{7}\right)^2 - 1 = \frac{2\times 9}{49} - 1$

$= \frac{18}{49} - 1$

$= \frac{-31}{49}$

105. (B)

106. (B) Standard deviation doesn't change on adding or subtracting of a number

$\therefore$ New standard deviation $= \frac{10}{20}$

$= 0.5$

107. (C) Required no. of students = 12 + 15 + 24

= 51

108. (D)

Class Interval	Frequency	c.f.
160 – 162	12	12
162 – 164	15	27
164 – 166	24	51
166 – 168	13	64
	N = 64	

We know that the median class in statistics is the class interval where the cumulative frequency first becomes greater than or equal to $\frac{N}{2}$.

Here $\frac{N}{2} < c \Rightarrow \frac{64}{2} < 51$

Hence, median class = 164 – 166

Required median $= l + \left(\frac{\frac{n}{2} - cf}{f}\right)\times h$

$= 164 + \left(\frac{32-15}{24}\right)\times 2$

$= 164 + \frac{17}{12}$

= 164 + 1.41 = 165.41 cm

109. (D) We have to find the height which occurs most frequently in the class it means that we have find mode. modal class = 164 – 166 because it has max. frequency.

Required height or mode

$= L + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right)\times h$

$= 164 + \left(\frac{24-15}{48-15-13}\right)\times 2$

$= 164 + \frac{9}{20}\times 2$

= 164 + 0.9 = 164.9 cm

110. (C) The most appropriate graphical representation of the given frequency distribution is histogram.

111. (B) $\text{Var}(X) = \frac{\sum X^2}{n} - \left(\frac{\sum X}{n}\right)^2$

$= \frac{900}{50} - \left(\frac{200}{50}\right)^2$

= 18 – 16 = 2

$\text{Var}(Y) = \frac{\sum Y^2}{n} - \left(\frac{\sum Y}{n}\right)^2$

$= \frac{1400}{50} - \left(\frac{250}{50}\right)^2$

= 28 – 25

= 3

Hence, variance (X) < variance (Y)

112. (A) Coefficient of variation of X = $\frac{\text{S.D}}{\text{X}}\times 100$

$= \frac{\sqrt{2}}{\left(\frac{200}{50}\right)}\times 100 = 25\sqrt{2}$

Coefficient of variation of

$Y = \frac{\text{S.D}}{\text{Y}}\times 100$

$= \frac{\sqrt{3}}{\left(\frac{250}{50}\right)}\times 100 = 20\sqrt{3}$

Hence, coefficient of variation of X is strictly more than coefficient of variation of Y.

113. (C) Given that

$P(A) = \frac{3}{10}$, $P(B) = \frac{1}{2}$ $P(C) = \frac{4}{5}$

and $P\left(\frac{\text{Bonus}}{A}\right) = \frac{4}{9}$

$P\left(\frac{\text{Bonus}}{B}\right) = \frac{2}{9}$

$P\left(\frac{\text{Bonus}}{C}\right) = \frac{1}{3}$

Required probability (P)

$= P\left(\frac{\text{Bonus}}{A}\right)\times P(A) + P\left(\frac{\text{Bonus}}{B}\right)\times P(B) + P\left(\frac{\text{Bonus}}{C}\right)\times P(C)$

$= \frac{4}{9}\times\frac{3}{10} + \frac{2}{9}\times\frac{1}{2} + \frac{1}{3}\times\frac{4}{5}$

$= \frac{2}{15} + \frac{1}{9} + \frac{4}{15} = \frac{6+5+12}{45} = \frac{23}{45}$

114. (A) Required Probability

$= \frac{P\left(\frac{\text{Bonus}}{B}\right)\cdot P(B)}{P(\text{Total Bonus})}$

$= \frac{\frac{2}{9}\times\frac{1}{2}}{\frac{23}{45}}$

$= \frac{1}{9}\times\frac{45}{23} = \frac{5}{23}$

115. (B) $P(A^C) = 1 - P(A)$

$= 1 - \frac{1}{3} = \frac{2}{3}$

$\therefore P(B \cap A^C) = P(B) - P(B \cap A)$

$= \frac{1}{2} - \frac{1}{4}$

$= \frac{1}{4}$

Required probability

$P(B/A^C) = \frac{P(B \cap A^C)}{P(A^C)}$

$$= \frac{\frac{1}{4}}{\frac{2}{3}} = \frac{3}{8}$$

116. (B) $P(A^C \cap B^C) = P(A \cup B)^C$

$= 1 - P(A \cup B)$

$= 1 - P(A) - P(B) + P(A \cap B)$

$= 1 - \frac{1}{3} - \frac{1}{2} + \frac{1}{4}$

$= \frac{5}{4} - \frac{5}{6} = \frac{15-10}{12} = \frac{5}{12}$

117. (B) If two fair dice are tossed then total outcomes = 36

then $\frac{N}{2} < C \Rightarrow \frac{64}{2} < 51$

Favourable outcomes = {(2, 6), (3, 5), (4, 4), (5, 3), (6, 2), (3, 6), (4, 5), (4, 6), (5, 4), (5, 5), (5, 6), (6, 3), (6, 4), (6, 5), (6, 6)}

total favourable outcomes = 15

Required probability = $\frac{15}{36} = \frac{5}{12}$

118. (C) Given that, $p = \frac{1}{5},\ q = \frac{4}{5}$

$n = 7$

Required probability = $1 - [p(n = 0) + p(n = 1)]$

$= 1 - \left[{}^7C_0 \left(\frac{4}{5}\right)^7 + {}^7C_1 \times \frac{1}{5} \times \left(\frac{4}{5}\right)^6\right]$

$= 1 - \left[\left(\frac{4}{5}\right)^7 + 7 \times \frac{1}{5}\left(\frac{4}{5}\right)^6\right]$

$= 1 - \left(\frac{4}{5}\right)^6 \left[\frac{4}{5} + \frac{7}{5}\right]$

$= 1 - \left(\frac{4}{5}\right)^6 \times \frac{11}{5}$

$= 1 - \left(\frac{11}{5}\right)\left(\frac{4}{5}\right)^6$

119. (C) Given that

$9\,P(X = 4) = P(X = 2)$

$9 \times {}^6C_4\, k^4(1-k)^2 = {}^6C_2\, k^2(1-k)^4$

$9 \times \frac{\lfloor 6}{\lfloor 4 \lfloor 2} \times k^2 = \frac{\lfloor 6}{\lfloor 2 \lfloor 4}(1-k)^2$

$9k^2 = (1-k)^2$

On taking square root both sides, we get

$3k = 1 - k$

$4k = 1$

$k = \frac{1}{4}$

120. (A) $P(X = 3) = {}^6C_3\, k^3(1-k)^3$

$= \frac{\lfloor 6}{\lfloor 3 \lfloor 3} \times \left(\frac{1}{4}\right)^3 \left(\frac{3}{4}\right)^3$

$= \frac{6 \times 5 \times 4}{3 \times 2 \times 1} \times \left(\frac{1}{4}\right)^3 \times \left(\frac{3}{4}\right)^3$

$= 20 \times \frac{27}{64 \times 64}$

$= \frac{135}{1024}$

❑❑