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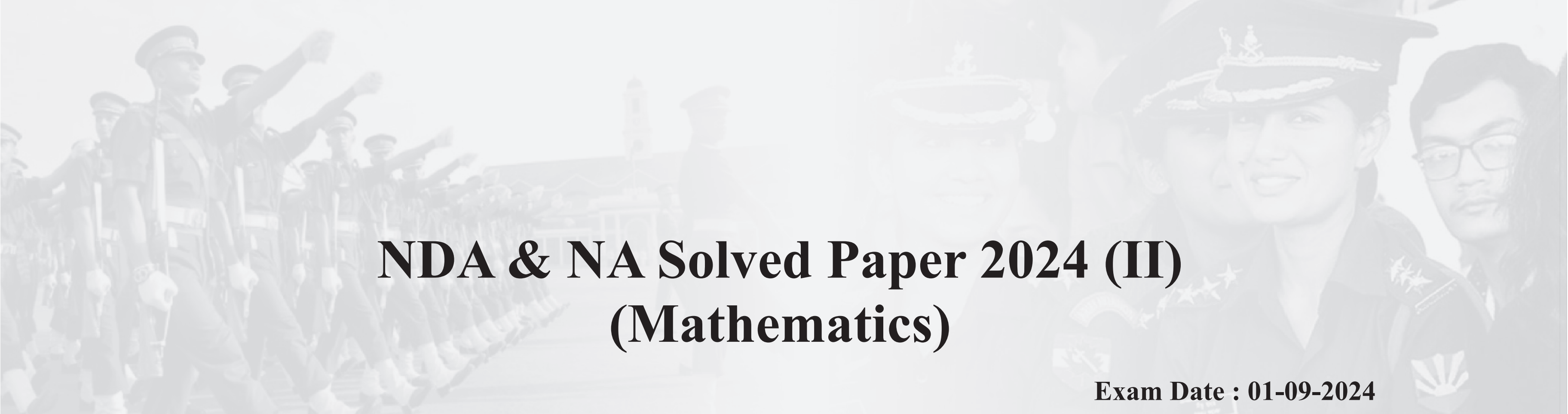


NDA & NA Solved Paper 2024 (II)
(Mathematics)

Exam Date : 01-09-2024

Set, Relations and Functions

1. Consider the following statements :

1. The set of all irrational numbers between $\sqrt{12}$ and $\sqrt{15}$ is an infinite set.
2. The set of all odd integers less than 1000 is a finite set.

Which of the statement given above is/are correct :

(A) 1 only (B) 2 only
(C) Both 1 and 2 (D) Neither 1 nor 2

2. Let P and Q be two non-void relations on a set A. Which of the following statements are correct ?

1. P and Q are reflexive $\Rightarrow P \cap Q$ is reflexive.
2. P and Q are symmetric $\Rightarrow P \cup Q$ is symmetric.
3. P and Q are transitive $\Rightarrow P \cap Q$ is transitive.

Select the answer using the code given below :

(A) 1 and 2 only (B) 2 and 3 only
(C) 1 and 3 only (D) 1, 2 and 3

3. If A and B are two non-empty sets having 10 elements in common, then how many elements do $A \times B$ and $B \times A$ have in common ?

(A) 10 (B) 20
(C) 40 (D) 100

4. In a class of 240 students, 180 passed in English, 130 passed in Hindi and 150 passed in Sanskrit. Further, 60 passed in only one subject, 110 passed in only two subjects and 10 passed in none of the subjects. How many passed in all three subjects ?

(A) 60 (B) 55
(C) 40 (D) 35

5. Let $z = [y]$ and $y = [x] - x$, where [.] is the greatest integer function. If x is not an integer but positive, then what is the value of z ?

(A) –1 (B) 0
(C) 1 (D) 2

6. If $f(x) = 4x + 1$ and $g(x) = kx + 2$ such that $fog(x) = gof(x)$, then what is the value of k ?

(A) 7 (B) 5
(C) 4 (D) 3

7. If $f(2x) = 4x^2 + 1$, then for how many real values of x will $f(2x)$ be the GM of $f(x)$ and $f(4x)$?

(A) Four (B) Two
(C) One (D) None

8. If $f(x) = [x]^2 - 30[x] + 221 = 0$, where $[x]$ is the greatest integer function, then what is the sum of all integer solutions.

(A) 13 (B) 17
(C) 27 (D) 30

9. If $f(x) = 9x - 8\sqrt{x}$ such that $g(x) = f(x) - 1$, then which one of the following is correct ?

(A) $g(x) = 0$ has no real roots
(B) $g(x) = 0$ has only one real root which is an integer.
(C) $g(x) = 0$ has two real roots which are integers
(D) $g(x) = 0$ has only one real root which is not an integer.

10. Let $f(x) f(y) = f(xy)$ for all real x, y, If $f(2) = 4$, then what is the value of $f(1/2)$?

(A) 1/4 (B) 1/2
(C) 1 (D) 4

Direction (Q. No. 11 and 12)

Consider the following for the two items that follow :

Let $fog(x) = \cos^2\sqrt{x}$ and $gof(x) = |\cos x|$.

11. Which one of the following is $f(x)$?

(A) $\cos x$ (B) $\cos x^2$
(C) $\cos^2 x$ (D) $\cos |x|$

12. Which one of the following is $g(x)$?

(A) $\sqrt{x}$ (B) $|x|$
(C) x^2 (D) $x|x|$

Logarithms and their Properties

13. What is the minimum value of the function $f(x) = \log_{10}(x^2 + 2x + 11)$?

(A) 0 (B) 1
(C) 2 (D) 10

Complex Number

14. If $\omega \neq 1$ is a cube root of unity, then what is $(1 + \omega - \omega^2)^{100} + (1 - \omega + \omega^2)^{100}$ equal to ?

(A) $2^{100}\omega^2$ (B) $2^{100}\omega$
(C) 2^{100} (D) -2^{100}

15. What is the value of the sum $\sum_{n=1}^{20}(i^{n-1} + i^n + i^{n+1})$ where $i = \sqrt{-1}$?

(A) $-2i$ (B) 0
(C) 1 (D) $2i$

Direction (Q. No. 16 and 17)

Consider the following for the two items that follow :

Let Z_1 and Z_2 be any two complex numbers such that $Z_1^2 + Z_2^2 + Z_1Z_2 = 0$

16. What is the value of $\left|\frac{Z_1}{Z_2}\right|$?

(A) 1 (B) 2
(C) 3 (D) 4

17. What is the value of $\frac{1}{2} + \text{Re}\left(\frac{Z_1}{Z_2}\right)$?

(A) –1 (B) 0
(C) 1 (D) 2

Theory of Equations and Inequalities

18. If n is a root of the equation $x^2 + px + m = 0$ and m is a root of the equation $x^2 + px + n = 0$, where $m \neq n$, then what is the value of $p + m + n$?

(A) –1 (B) 0
(C) 1 (D) 2

19. What is the number of real roots of the equation $(x - 1)^2 + (x - 3)^2 + (x - 5)^2 = 0$?

(A) None (B) Only one
(C) Only two (D) Three

Sequences and Series

20. If p times the pth term of an AP is equal to q times the qth term $(p \neq q)$, then what is the $(p + q)$th term equal to ?

(A) 0 (B) $p + q$
(C) pq (D) $pq(p + q)$

21. Let $p = \ln(x)$, $q = \ln(x^3)$ and $r = \ln(x^5)$, where $x > 1$. Which of the following statements is/are correct ?

I. p, q and r are in AP.

II. p, q and r can never be in GP.

Select the answer using the code given below.

(A) I only (B) II Only

(C) Both I and II (D) Neither I and II

22. Let $x > 1$, $y > 1$, $z > 1$ be in GP. Then $\frac{1}{1+\ln x}, \frac{1}{1+\ln y}, \frac{1}{1+\ln z}$ are :

(A) in A.P.

(B) in G.P.

(C) in H.P.

(D) neither in AP nor in GP nor in HP

23. If the sum of the first n terms of a series is $n(2n + 1)$, then what is the nth term ?

(A) $4n - 1$ (B) $4n$

(C) $4n + 1$ (D) $4n + 3$

24. In an AP, the ratio of the sum of the first p terms to the sum of the first q terms is $p^2 : q^2$, Which one of the following is correct ?

(A) The first term is equal to the common difference.

(B) The first term is equal to twice the common difference.

(C) The common difference is equal to twice the first term.

(D) The first term is equal to square of the common difference.

Direction (Q. No. 25 and 26)

Consider the following for the two items that follow :

The product of 5 consecutive terms of an AP is 229635. The first, second and fifth terms are in GP.

25. What is the common difference ?

(A) 3 (B) 4

(C) 5 (D) 6

26. What is the sum of all five terms ?

(A) 60 (B) 65

(C) 75 (D) 80

Direction (Q. No. 27 and 28)

Consider the following for the two items that follow :

The roots of the quadratic equation $a^2(b^2 - c^2)x^2 + b^2(c^2 - a^2)x + c^2(a^2 - b^2) = 0$ are equal ($a^2 \neq b^2 \neq c^2$).

27. Which one of the following statements is correct ?

(A) a^2, b^2, c^2 are in AP

(B) a^2, b^2, c^2 are in GP

(C) a^2, b^2, c^2 are in HP

(D) a^2, b^2, c^2 are neither in AP nor in GP nor in HP

28. Which one of the following is a root of the equation ?

(A) $\frac{b^2(c^2 - a^2)}{a^2(c^2 - b^2)}$ (B) $\frac{b^2(c^2 - a^2)}{a^2(b^2 - c^2)}$

(C) $\frac{b^2(c^2 - a^2)}{2a^2(c^2 - b^2)}$ (D) $\frac{b^2(c^2 - a^2)}{2a^2(b^2 - c^2)}$

Permutations and Combinations

29. How many 4-digit numbers are there having all digits as odd ?

(A) 625 (B) 400

(C) 196 (D) 120

30. In how many ways can the letters of the word INDIA be permutated such that in each combination, vowels should occupy odd positions ?

(A) 3 (B) 6

(C) 9 (D) 12

31. The letters of the word EQUATION are arranged in such a way that all vowels as well as consonants are together. How many such arrangements are there ?

(A) 240 (B) 720

(C) 1440 (D) 1620

32. In how many ways can a student choose $(n - 2)$ courses out of n courses if 2 courses are compulsory ($n > 4$) ?

(A) $(n - 3)(n - 4)$ (B) $(n - 1)(n - 2)$

(C) $(n - 3)(n - 4)/2$ (D) $(n - 2)(n - 3)/2$

33. Three perfect dice D_1, D_2 and D_3 are rolled. Let x, y and z represent the numbers on D_1, D_2 and D_3 respectively. What is the number of possible outcomes such that $x < y < z$?

(A) 20 (B) 18

(C) 14 (D) 10

Binomial Theorem and Its Applications

34. In the expansion of $(1 + x)^p (1 + x)^q$, if the coefficient of x^3 is 32, then what is the value of $(p + q)$?

(A) 5 (B) 6

(C) 7 (D) 8

35. What is the remainder when $7^n - 6n$ is divided by 36 for $n = 100$?

(A) 0 (B) 1

(C) 2 (D) 6

36. What is V + W equal to ?

(A) 8 (B) 4

(C) 2 (D) 1

37. What is the value of (U + V)W ?

(A) 1/2 (B) 1

(C) 3/2 (D) 2

Trigonometry

38. The roots of the equation $7x^2 - 6x + 1 = 0$ are $\tan \alpha$ and $\tan \beta$, where 2α and 2β are the angles of a triangle. Which one of the following is correct ?

(A) The triangle is equilateral

(B) The triangle is isosceles but not right-angled

(C) The triangle is right-angled

(D) The triangle is right-angled isosceles

39. What is the number of solutions of the equation $\cot 2x . \cot 3x = 1$ for $0 < x < \pi$?

(A) only one (B) only two

(C) only five (D) More than five

40. What is the general solution of $\cos^{100} x - \sin^{100} x = 1$?

(A) $n\pi$ (B) $(2n + 1)\pi$

(C) $2n\pi$ (D) $(2n + 1)\pi/2$

where n is an integer.

41. What is $\sin 12° \sin 48°$ equal to ?

(A) $\frac{\sqrt{5}-1}{4}$ (B) $\frac{\sqrt{5}+1}{4}$

(C) $\frac{\sqrt{5}-1}{8}$ (D) $\frac{\sqrt{5}+1}{8}$

42. What is $\frac{\cos 17° - \sin 17°}{\cos 17° + \sin 17°}$ equal to ?

(A) tan 34° (B) cot 34°

(C) tan 62° (D) cot 62°

43. Consider the following numbers :

1. tan 22.5°
2. cot 22.5°
3. tan 22.5° – cot 22.5°

(A) None (B) only one

(C) only two (D) All three

44. If $\frac{x}{\cos\theta} = \frac{y}{\cos\left(\frac{2\pi}{3} - \theta\right)} = \frac{z}{\cos\left(\frac{2\pi}{3} + \theta\right)}$

then what is $x + y + z$ equal to ?

(A) –1 (B) 0

(C) 1 (D) 3

45. If $p \tan(\theta - 30°) = q \tan(\theta + 120°)$, then what is $(p + q) / (p - q)$ equal to ?

(A) $\sin 2\theta$ (B) $\cos 2\theta$

(C) $2\sin 2\theta$ (D) $2\cos 2\theta$

46. What is the maximum value of $a\cos x + b \sin x + c$?

(A) $\sqrt{a^2 + b^2 + c}$ (B) $\sqrt{a^2 + b^2} + c$

(C) $\sqrt{a^2 + b^2} - c$ (D) $\sqrt{a^2 + b^2}$

Properties of Triangle

47. In a triangle ABC $\frac{a}{\cos A} = \frac{b}{\cos B} = \frac{c}{\cos C}$

What is the area of the triangle if a = 6 cm ?

(A) $9\sqrt{3}$ square cm

(B) 12 square cm

(C) $18\sqrt{3}$ square cm

(D) 24 square cm

48. In a triangle ABC, $\angle A = 75°$ and $\angle B = 45°$. What is $2a - b$ equal to ?

(A) c (B) $\sqrt{2}c$

(C) $2c$ (D) $2\sqrt{2}c$

49. In a triangle ABC tan A + tan B + tan C = k. What is the value of cot A cot B cot C ?

(A) $0.5k$ (B) $1/k$

(C) $3/k$ (D) $1/k^3$

Inverse Trigonometric Functions

50. If $4\sin^{-1} x + \cos^{-1} x = \pi$, then what is $\sin^{-1} x + 4\cos^{-1} x$ equal to ?

(A) $\pi/2$ (B) π

(C) $3\pi/2$ (D) 2π

51. What is $\cot^2(\sec^{-1} 2) + \tan^2(\text{cosec}^{-1} 3)$ equal to ?

(A) 11/12 (B) 11/24

(C) 7/24 (D) 1/24

Matrix

52. Let X be a matrix of order 3 × 3, Y be a matrix of order 2 × 3 and Z be a matrix of order 3 × 2. Which of the following statements are correct ?

1. (ZY)X is defined and is a square matrix of order 3.
2. Y(XZ) is defined and is a square matrix of order 2.
3. X (YZ) is not defined.

Select the answer using the code given below.

(A) only 1 and 2 (B) only 2 and 3

(C) only 1 and 3 (D) 1, 2 and 3

53. Consider the following in respect of the matrices.

$$P = \begin{bmatrix} 0 & c & -b \\ -c & 0 & a \\ b & -a & 0 \end{bmatrix}$$

and $$Q = \begin{bmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{bmatrix}$$

I. PQ is a null matrix

II. QP is identity matrix of order 3.

III. PQ = QP

Which of the above, is/are correct ?

(A) 1 only (B) 1 and 2

(C) 1 and 3 (D) 2 and 3

Determinants

54. Let A and B be two square matrices of same order. If AB is a null matrix, then which one of the following is correct ?

(A) Both A and B are null matrices

(B) Either A or B is a null matrix

(C) B is a null matrix if A is a non-singular matrix

(D) Both A and B are singular matrices

55. If $Z = \frac{1}{3}\begin{vmatrix} i & 2i & 1 \\ 2i & 3i & 2 \\ 3 & 1 & 3 \end{vmatrix} = x + iy; i = \sqrt{-1}$ then what is modulus of Z equal to ?

(A) 1 (B) $\sqrt{2}$

(C) 2 (D) $\sqrt{3}$

56. If $\omega = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$ then what is

$$\begin{vmatrix} 1+\omega & 1+\omega^2 & \omega+\omega^2 \\ 1 & \omega & \omega^2 \\ \frac{1}{\omega} & \frac{1}{\omega^2} & 1 \end{vmatrix}$$ equal to

(A) 0 (B) ω

(C) ω^2 (D) $1 - \omega^2$

57. If $D_n = \begin{vmatrix} n & 20 & 30 \\ n^2 & 40 & 50 \\ n^3 & 60 & 70 \end{vmatrix}$ then what is the value of $\sum_{n=1}^{4} D_n$?

(A) –10000 (B) –10

(C) 10 (D) 10000

58. If P is a skew-symmetric matrix of order 3, then what is det(P) equal to ?

(A) –1 (B) 0

(C) 1 (D) 3

Direction (Q. No. 59 and 60)

Consider the following for the two items that follow :

Let $$A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$$

59. What is A(adj A) equal to ?

(A) $\begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

(B) $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

(C) $\begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1/2 \end{bmatrix}$

(D) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

60. What is A^{-1} equal to ?

(A) $\begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}$

(B) $\begin{bmatrix} 1/2 & -1/2 & 0 \\ -1 & 3/2 & -2 \\ -1 & 3/2 & -3/2 \end{bmatrix}$

(C) $\begin{bmatrix} 2 & -2 & 0 \\ -4 & 6 & -8 \\ -4 & 6 & -6 \end{bmatrix}$

(D) $\begin{bmatrix} 1/5 & -1/5 & 0 \\ -2/5 & 3/5 & -4/5 \\ -2/5 & 3/5 & -3/5 \end{bmatrix}$

Straight Lines

61. What is the maximum number of possible points of intersection of four straight lines and a circle (intersection is between lines as well as circle and lines) ?

(A) 6 (B) 10

(C) 14 (D) 16

62. The diagonals of a quadrilateral ABCD are along the lines $x - 2y = 1$ and $4x + 2y = 3$. The quadrilateral ABCD may be a :

(A) rectangle

(B) cyclic quadrilateral

(C) parallelogram

(D) rhombus

63. If P(2, 4), Q (8, 12), R(10, 14) and S(x, y) are vertices of a parallelogram, then what is $(x + y)$ equal to ?

(A) 8 (B) 10

(C) 12 (D) 14

64. What are the coordinates of vertex D?

(A) (2, 1) (B) (1, 2)

(C) (1, 1) (D) (3, 1)

65. What is the point of intersection of the diagonals of the trapezium ?

(A) (3, 7 / 2) (B) (3, 7 / 3)

(C) (7 / 2, 2) (D) (5/2, 2)

Conic Section

66. The foci of the ellipse $4x^2 + 9y^2 = 1$ are at Q and R. If P(x, y) is any point on the ellipse, then what is PQ + PR equal to?

(A) 2 (B) 1
(C) 2/3 (D) 1/3

67. The equation of a circle is.
$(x^2 - 4x + 3) + (y^2 - 6y + 8) = 0$
Which of the following statements are correct ?
1. The end points of a diameter of the circle are at (1, 2) and (3, 4).
2. The end points of a diameter of the circle are at (1, 4) and (3, 2).
3. The end points of a diameters of the circle are at (2, 4) and (4, 2).

Select the Answer using the code given below.
(A) only 1 and 2 (B) only 2 and 3
(C) only 1 and 3 (D) 1, 2 and 3

68. Consider the points P(4k, 4k) and Q(4k, –4k) lying on the parabola $y^2 = 4kx$. If the vertex is A, then what is ∠PAQ equal to?
(A) 60° (B) 90°
(C) 120° (D) 135°

Direction (Q. No. 69 and 70)
Consider the following for the two items that follow :

A triangle ABC is inscribed in the circle $x^2 + y^2 = 100$. B and C have coordinates (6, 8) and (–8, 6) respectively.

69. What is ∠BAC equal to?
(A) $\pi/2$ (B) $\pi/3$ or $2\pi/3$
(C) $\pi/4$ or $3\pi/4$ (D) $\pi/6$ or $5\pi/6$

70. What are the coordinates of A ?
(A) (–6, 8)
(B) (–6, –8)
(C) $(5,\sqrt{2},5\sqrt{2})$
(D) Cannot be determined due to insufficient data

Limits, Continuity and Differentiability

71. Which one of the following is correct regarding $\lim_{x\to3}\frac{|x-3|}{x-3}$?
(A) Limit exists and is equal to 1
(B) Limit exists and is equal to 0
(C) Limit exists and is equal to –1
(D) Limit does not exist

72. What is $\lim_{\theta\to\frac{\pi}{2}}$ (sec θ – tan θ) equal to ?
(A) –1 (B) 0
(C) 1/2 (D) 1

Direction (Q. No. 73 and 74)
Consider the following for the next two items that follow :

Let $f(x) = [x]^2 - [x^2]$.

73. What is $f(0.999) + f(1.001)$ equal to?
(A) –1 (B) 0
(C) 1 (D) 2

74. Consider the following statements :
1. $f(x)$ is continuous at $x = 0$.
2. $f(x)$ is continuous at $x = 1$.

Which of the statements given above is/are correct ?
(A) 1 only (B) 2 only
(C) Both 1 and 2 (D) Neither 1 and 2

Direction (Q. No. 75 and 76)
Consider the following for the two items that follow :

Let $f: (-1, 1) \to R$ be a differentiable function with $f(0) = -1$ and $f'(0) = 1$. Let $h(x) = f(2f(x) + 2)$ and $g(x) = (h(x))^2$.

75. What is the $h'(0)$ equal to?
(A) –2 (B) –1
(C) 0 (D) 2

76. What is the $g'(0)$ equal to?
(A) –4 (B) –2
(C) 0 (D) 4

Application of Differentiation

Direction (Q. No. 77 and 78)
Consider the following for the two items that follow :

Let $f(x) = \cos 2x + x$ on $[-\pi/2, \pi/2]$.

77. What is the greatest value of $f(x)$?
(A) $\frac{\sqrt{3}}{2}-\frac{\pi}{12}$ (B) $\frac{\sqrt{3}}{2}+\frac{\pi}{12}$
(C) $\frac{\sqrt{3}}{2}+\frac{\pi}{9}$ (D) $\frac{\sqrt{3}}{2}+\frac{\pi}{6}$

78. What is the least value of $f(x)$?
(A) $-\left(1+\frac{\pi}{2}\right)$ (B) $-\left(\frac{1}{2}+\frac{\pi}{2}\right)$
(C) $-\left(1+\frac{\pi}{4}\right)$ (D) $-2\left(\frac{1}{2}-\frac{\pi}{4}\right)$

Indefinite Integration

Direction (Q. No. 79 and 80)
Consider the following for the two items that follow:

Let $2\int\frac{x^2-1}{\sqrt{x^2+1}}dx$ = U(x) V(x) – 3 ln {U(x) + V(x)} + c

79. What is $|U^2(x) - V^2(x)|$ equal to ?
(A) 0 (B) 1
(C) 2 (D) 3

80. What is U(x) V(x) equal to ?
(A) $\sqrt{x^2+x^4}$ (B) $\sqrt{x+x^3}$
(C) $\frac{\sqrt{x^2+x^4}}{2}$ (D) $2\sqrt{x^2+x^4}$

Definite Integration

Direction (Q. No. 81 and 82)
Consider the following for the two items that follow:

Let $f(x) = |x^2 - x - 2|$.

81. What is $\int_0^2 f(x)dx$ equal to ?
(A) 0 (B) 1
(C) 5/3 (D) 10/3

82. What is $\int_1^3 f(x)dx$ equal to ?
(A) 2 (B) 3
(C) 4 (D) 5

Direction (Q. No. 83 and 84)
Consider the following for the two items that follow :

Let $f(t) = \ln\left(t+\sqrt{1+t^2}\right)$ and $g(t) = \tan(f(t))$.

83. Consider the following statements:
1. $f(f)$ is an odd function.
2. $g(t)$ is an odd function.

Which of the statements given above is/are correct ?
(A) 1 only (B) 2 only
(C) Both 1 and 2 (D) Neither 1 and 2

84. What is $\int_{-\pi}^{\pi} g(t)\,dt$ equal to?
(A) –1 (B) 0
(C) 1/2 (D) 1

Direction (Q. No. 85 and 86)
Consider the following for the two items that follow:

Let $I = \int_0^{\pi/2}\frac{f(x)}{g(x)}dx$, where $f(x) = \sin x$ and $g(x) = \sin x + \cos x + 1$.

85. What is $\int_0^{\pi/2}\frac{dx}{g(x)}$ equal to ?
(A) $\frac{\ln 2}{2}$ (B) $\frac{\ln 2}{4}$
(C) ln 2 (D) 2 ln 2

86. What is I equal to ?
(A) $\frac{\pi}{4}+\ln 2$ (B) $\frac{\pi}{4}-\ln 2$
(C) $\frac{\pi}{4}-\frac{\ln 2}{2}$ (D) $\frac{\pi}{4}+\frac{\ln 2}{2}$

Application of Integration

Direction (Q. No. 87 and 88)
Consider the following for the next **two (02)** items that follow:

The area bounded by the parabola $y^2 = kx$ and the line $x = k$, where $k > 0$, is 4/3 square units.

87. What is the value of k ?

(A) 1/2 (B) 1

(C) $\sqrt{2}$ (D) 2

88. What is the area of the parabola bounded by the latus rectum ?

(A) 1/6 square unit (B) 2/3 square unit

(C) 1 square unit (D) 4/3 square unit

Differential Equations

Direction (Q. No. 89 and 90)

Consider the following for the two items that follow:

Let $y\,dx + (x - y^3)\,dy = 0$ be a differential equation.

89. What are the order and degree respectively of the differential equation ?

(A) 1 and 1 (B) 1 and 2

(C) 2 and 1 (D) 1 and 3

90. What is the solution of the differential equation?

(A) $y^4 + 2x = c$ (B) $y^4 + 3x = c$

(C) $2xy^4 + x = c$ (D) $4xy - y^4 = c$

Vector Algebra

91. What is $3\alpha+2\beta$ equal to if $(2\hat{i} + 6\hat{j} + 27\hat{k}) \times (\hat{i} + \alpha\hat{j} + \beta\hat{k})$ is a null vector?

(A) 36 (B) 33

(C) 30 (D) 27

92. For what value of the angle between the vectors $\vec{a}$ and $\vec{b}$ is the quantity $\left|\vec{a}\times\vec{b}\right| + \sqrt{3}\,|\vec{a}\cdot\vec{b}|$ maximum ?

(A) 0° (B) 30°

(C) 45° (D) 60°

93. Let θ be the angle between two unit vectors $\vec{a}$ and $\vec{b}$. If $\vec{a}+2\vec{b}$ is perpendicular to $5\vec{a}-4\vec{b}$, then what is $\cos\theta + \cos2\theta$ equal to ?

(A) 0 (B) 1 / 2

(C) 1 (D) $\frac{\sqrt{3}+1}{2}$

94. Let ABCDEF be a regular hexagon. If $\overrightarrow{AD} = m\,\overrightarrow{BC}$ and $\overrightarrow{CF} = n\,\overrightarrow{AB}$, then what is mn equal to ?

(A) –4 (B) –2

(C) 2 (D) 4

95. The vectors $\vec{a},\vec{b}$ and $\vec{c}$ are of the same length. If taken pairwise, they form equal angles. If $\vec{a} = \hat{i} + \hat{j}$ and $\vec{b} = \hat{j} + \hat{k}$, then what can $\vec{c}$ be equal to ?

1. $\hat{i} + \hat{k}$

2. $\frac{-\hat{i} + 4\hat{j} - \hat{k}}{3}$

Select the correct answer using the code given below.

(A) 1 only (B) 2 only

(C) Both 1 and 2 (D) Neither 1 nor 2

3D Geometry

Direction (Q. No. 96 and 97)

Consider the following for the next **two (02)** items that follow:

Let $2x^2 + 2y^2 + 2z^2 + 3x + 3y + 3z - 6 = 0$ be a sphere.

96. What are the diameter of the sphere ?

(A) $\frac{5\sqrt{3}}{4}$ (B) $\frac{5\sqrt{3}}{2}$

(C) $\frac{3\sqrt{5}}{4}$ (D) $\frac{3\sqrt{5}}{2}$

97. The centre of the sphere lies on the plane.

(A) $2x + 2y + 2z - 3 = 0$

(B) $4x + 4y + 4z - 3 = 0$

(C) $4x + 8y + 8z - 15 = 0$

(D) $4x + 8y + 8z + 15 = 0$

Direction (Q. No. 98 and 99)

Consider the following for the two items that follow :

Let S be the line of intersection of two planes $x + y + z = 1$ and $2x + 3y - 4z = 8$.

98. Which of the following are the direction rations or S ?

(A) (–7, –6, 1) (B) (–7, 6, 1)

(C) (–6, 5, 1) (D) (6, 5, 1)

99. If (l, m, n) are direction cosines of S, then what is the value of $43(l^2 - m^2 - n^2)$?

(A) 6 (B) 5

(C) 4 (D) 1

Direction (Q. No. 100 and 101)

Consider the following for the two items that follow:

Let L : $x + y + z + 4 = 0 = 2x - y - z + 8$ be a line and P : $x + 2y + 3z + 1 = 0$ be a plane.

100. What are the direction rations of the line ?

(A) (2, 1, –1) (B) (0, –1, 2)

(C) (0, 1, –1) (D) (2, 3, –3)

101. What is the point of intersection of L and P II ?

(A) (4, 3, –3) (B) (4, –3, 3)

(C) (–4, –3, –3) (D) (–4, –3, 3)

Statistics

102. Let $x-3y+4=0$ and $2x-7y+8=0$ be two lines of regression computed from some bivariate data. If b_{yx} and b_{xy} are regression coefficients of lines of regression of y on x and x on y respectively, then what is the value of $b_{xy} + 7b_{yx}$?

(A) –2 (B) 1

(C) 2 (D) 5

103. The mean of n observations 1, 4, 9, 16, ..., n^2 is 130. What is the value of n ?

(A) 18 (B) 19

(C) 20 (D) 21

104. What is the mean deviation of the first 10 natural numbers ?

(A) 2 (B) 2.5

(C) 3 (D) 3.5

105. Let $\sum_{i=1}^{9} x_i^2 = 855$. If M is the mean and σ is the standard deviation of $x_1, x_2, ..., x_9$, then what is the value of $M^2 + \sigma^2$?

(A) 100 (B) 95

(C) 90 (D) 85

106. The mean of the series $x_1, x_2, ..., x_n$ is $\bar{x}$. If x_n is replaced by k, then what is the new mean ?

(A) $\bar{x} - x_n + k$ (B) $\frac{n\bar{x} - \bar{x} + k}{n}$

(C) $\frac{\bar{x} - x_n - k}{n}$ (D) $\frac{n\bar{x} - x_n + k}{n}$

107. In a binomial distribution, if the mean is 6 and the standard deviation is $\sqrt{2}$, then what are the values of the parameters n and p respectively ?

(A) 18 and 1/3 (B) 9 and 1/3

(C) 18 and 2/3 (D) 9 and 2/3

Probability

108. Three distinct natural numbers are chosen at random from 1 to 10. What is the probability that they are consecutive ?

(A) 1/12 (B) 3/40

(C) 1/15 (D) 7/120

109. A, B, C and three mutually exclusive and exhaustive events associated with a random experiment. If 3P(B) = 4P(A) and 3P(C) = 2P(B), then what is P(A) equal to ?

(A) 7/29 (B) 8/29

(C) 9/29 (D) 10/29

110. A die has two faces with number 4, three faces with number 5 and one face with number 6. If the die is rolled once, then what is the probability of getting 4 or 5 ?

(A) $\frac{1}{3}$ (B) $\frac{2}{3}$

(C) $\frac{5}{6}$ (D) $\frac{1}{2}$

111. A box contains 2 black, 4 yellow and 6 white balls. Three balls are drawn in succession with replacement. What is the probability that all three are of the same colour ?

(A) $\frac{1}{6}$ (B) $\frac{1}{36}$

(C) $\frac{1}{12}$ (D) $\frac{5}{12}$

112. A can hit a target 5 times in 6 shots, B can hit 4 times in 5 shots and C can hit 3 times in 4 shots. What is the probability that A and C may hit but B may lose ?

(A) $\frac{1}{8}$ (B) $\frac{1}{6}$

(C) $\frac{1}{4}$ (D) $\frac{1}{3}$

113. The letters of the word ZOOLOGY are arranged in all possible ways. What is the probability that the consonants and vowels occur alternative ?

(A) $\frac{6}{35}$ (B) $\frac{3}{35}$

(C) $\frac{2}{35}$ (D) $\frac{1}{35}$

114. A natural number x is chosen at random from the first 100 natural numbers. What is the probability that $x^2 + x > 50$?

(A) $\frac{93}{100}$ (B) $\frac{47}{50}$

(C) $\frac{24}{25}$ (D) $\frac{23}{25}$

115. A fair coin is tossed till two heads occur in succession. What is the probability that the number of tosses required is less than 6 ?

(A) $\frac{5}{64}$ (B) $\frac{15}{32}$

(C) $\frac{31}{64}$ (D) $\frac{19}{32}$

116. Urn A contains 2 white and 2 black balls while urn B contains 3 white and 2 black balls. One ball is transferred from urn A to urn B and then a ball is drawn out of urn B. What is the probability that the ball is white ?

(A) $\frac{11}{20}$ (B) $\frac{7}{12}$

(C) $\frac{3}{5}$ (D) 1

117. For two events A and B, P(A) = P(A|B) = 0.25 and P(B/A) = 0.5. Which of the the following are correct ?

1. A and B are independent.
2. $P(A^c \cup B^c) = 0.875$
3. $P(A^c \cap B^c) = 0.375$

Select the answer using the code given below.

(A) only 1 and 2 (B) only 2 and 3

(C) only 1 and 3 (D) 1, 2 and 3

118. Two perfect dice are thrown. What is the probability that the sum of the numbers on the faces is neither 9 nor 10 ?

(A) $\frac{1}{36}$ (B) $\frac{5}{36}$

(C) $\frac{7}{36}$ (D) $\frac{29}{36}$

119. The occurrence of a disease in an industry is such that the workers have 20% chance of suffering from it. What is the probability that out of 6 workers chosen at random, 4 or more will suffer from the disease ?

(A) $\frac{53}{3125}$ (B) $\frac{63}{3125}$

(C) $\frac{73}{3125}$ (D) $\frac{83}{3125}$

120. Three perfect dice are rolled. Under the condition that no two show the same face, what is the probability that one of the faces shown is an ace (one) ?

(A) $\frac{5}{9}$ (B) $\frac{2}{3}$

(C) $\frac{1}{3}$ (D) $\frac{1}{2}$

Solutions

1. (A) $\sqrt{12}$ and $\sqrt{15}$ is an irrational numbers.

These are infinitely many irrational numbers between two irrational numbers.

∴ Statement 1 is correct. But statement 2 is incorrect as there infinite odd integer less than 1000 *i.e.*,

[...–3, –1, 1, 3, ... 999]

2. (A) If P and Q, both are reflexive, symmetry and transitive, then $P \cap Q$ is also reflexive, symmetry and transitive and $P \cup Q$ is reflexive and symmetry but not transitive.

∴ All the given statements are correct.

3. (D) If there are n elements common in set A and B, then n^2 elements are common in set A × B and B × A

So, Number of element in A × B and B × A = 10^2 = 100

4. (A) Total Number of students = Number of students passed in only one subject + Number of students passed in only two subject + Number of students passed in all three subjects + Number of students passed in none of the subjects.

Let n(A) belongs to the number of students passed in all three subjects

$\Rightarrow$ 240 = 60 + 110 + n(A) + 10

$\Rightarrow n$(A) = 240 – 180 = 60

5. (A) $0 < x - [x] < 1$ $\forall x$ is not an integer but positive

$-1 < [x] - x < 0$

$\Rightarrow -1 < y < 0$

$\Rightarrow [y] = -1$

$\therefore$ $z = -1$

6. (A) $f(x) = 4x + 1$ and $g(x) = kx + 2$

$go f(x) = fog\ (x)$

$\Rightarrow$ $g(4x + 1) = f(kx + 2)$

$\Rightarrow k(4x + 1) + 2 = 4(kx + 2) + 1$

$\Rightarrow$ $4kx + k + 2 = 4kx + 8 + 1$

$\Rightarrow$ $k = 7$

7. (C) If $f(2x) = 4x^2 + 1$

$= (2x)^2 + 1$

$f(x) = x^2 + 1$

$\Rightarrow$ $(f(2x))^2 = f(x) \times f(4x)$

[$\because f(2x)$ is GM of $f(x)$ and $f(4x)$]

$\Rightarrow (4x^2 + 1)^2 = (x^2 + 1)\ (16x^2 + 1)$

$\Rightarrow 16x^4 + 8x^2 + 1 = 16x^4 + 17x^2 + 1$

$\Rightarrow$ $9x^2 = 0$

$\Rightarrow$ $x = 0$

∴ There is only one real value of x.

8. (D) $f(x) = [x]^2 - 30[x] + 221 = 0$

$\Rightarrow ([x] - 17)\ ([x] - 13) = 0$

$\Rightarrow [x] = 17$ and $[x] = 13$

$\Rightarrow$ Sum of all integer solution

= 17 + 13 = 30

9. (B) $f(x) = 9x - 8\sqrt{x}$

$g(x) = f(x) - 1$

$= 9x - 8\sqrt{x} - 1$

$= 9x - 9\sqrt{x} + \sqrt{x} - 1$

$= 9\sqrt{x}\ (\sqrt{x} - 1) + 1(\sqrt{x} - 1)$

$= (9\sqrt{x} + 1)\ (\sqrt{x} - 1)$

$g(x) = 0$

$\Rightarrow (9\sqrt{x} + 1)(\sqrt{x} - 1) = 0$

$\Rightarrow \sqrt{x} = 1$

[as $\sqrt{x} = \frac{-1}{9}$ is not possible]

$\Rightarrow x = 1$

$\therefore g(x) = 0$ has only one real roots, which is an integer.

10. (A) $f(x) f(y) = f(xy)$

$f(2) = 4$

Let $x = 2 = y$, then

$f(2) + (2) = f(2 \times 2)$

$f(4) = 16$

Now, let $x = \frac{1}{2}$ and $y = 4$, then

$$f\left(\frac{1}{2}\right) \times f(4) = f\left(\frac{1}{2} \times 4\right)$$

$$f\left(\frac{1}{2}\right) \times 16 = 4$$

$$f\left(\frac{1}{2}\right) = \frac{1}{4}$$

11. (C) $fog(x) = \cos^2 \sqrt{x}$

$f[g(x)] = \cos^2 (\sqrt{x})$

$\Rightarrow f(x) = \cos^2 x$ and $g(x) = \sqrt{x}$

$gof(x) = \sqrt{\text{Cos}^2 x} = |\cos x|$

$\therefore f(x) = \cos^2 x$

12. (A) $g(x) = \sqrt{x}$

13. (B) $f(x) = \log_{10}(x^2 + 2x + 11)$

$$= \frac{\log(x^2 + 2x + 11)}{\log_{10}}$$

$f'(x)$

$$= \frac{1}{\log_{10}} \times \frac{1}{x^2 + 2x + 11} \times (2x + 2)$$

$f'(x) = 0$

$\Rightarrow x = -1$

$f''(x)$

$$= \frac{1}{\log_{10}}\left[\frac{(x^2 + 2x + 11)2 - (2x + 2)(2x + 2)}{(x^2 + 2x + 11)^2}\right]$$

$$= \frac{-2x^2 - 4x + 18}{\log_{10}(x^2 + 2x + 11)^2}$$

$$f''(-1) = \frac{-2 + 4 + 18}{\log_{10} \times (1 + 2 + 11)^2} > 0$$

$f(x)$ has minimum value at $x = -1$

$f(-1) = \log_{10} (x^2 + 2x + 11)$

$= \log_{10} (1 - 2 + 11)$

$= 1$

14. (D) $(1 + \omega - \omega^2)^{100} + (1 - \omega + \omega^2)^{100}$

$= (-\omega^2 - \omega^2)^{100} + (-\omega - \omega)^{100}$

$[\because 1 + \omega + \omega^2 = 0]$

$= (-2\omega^2)^{100} + (-2\omega)^{100}$

$= (-2)^{100} [(\omega^2)^{100} + \omega^{100}]$

$= (-2)^{100} [\omega^{200} + \omega^{100}]$

$= (-2)^{100} [\omega^2 + \omega]$ $[\because \omega^{3n} = 1]$

$= (-2)^{100} (-1)$ $[\because 1 + \omega + \omega^2 = 0]$

$= -(-2)^{100}$

$= -2^{100}$

15. (B) $\sum_{n=1}^{20} (i^{n-1} + i^n + i^{n+1})$

$$= \sum_{n=1}^{20} (i^{n-1} + i^n + i^{n+1} + i^{n+2})$$

$$- \sum_{n=1}^{20} i^{n+2}$$

As sum of 4 consecutive powers of i in 0, $i^{n-1} + i^n + i^{n+1} + i^{n+2} = 0$

$= 0 - (i^3 + i^4 + i^5 + i^6 + i^7 + i^8 + .. + i^{22})$

$= 0$

16. (A) $Z_1^2 + Z_2^2 + Z_1Z_2 = 0$

$$\Rightarrow \left(\frac{Z_1}{Z_2}\right)^2 + 1 + \frac{Z_1}{Z_2}$$

Let $\frac{Z_1}{Z_2}$ be Z

$\Rightarrow Z^2 + Z + 1 = 0$

$\Rightarrow Z = \omega, \omega^2$

$$|Z| = \left|\frac{Z_1}{Z_2}\right| = 1$$

17. (B) $Z = \omega, \omega^2 = \frac{-1 + \sqrt{3}i}{2}$

$$\text{Re}(Z) = \text{Re}\left(\frac{Z_1}{Z_2}\right) = \frac{-1}{2}$$

$$\Rightarrow \frac{1}{2} + \text{Re}\left(\frac{Z_1}{Z_2}\right) = \frac{1}{2} - \frac{1}{2} = 0$$

18. (C) As n is the root of $x^2 + px + m = 0$,

$n^2 + pn + m = 0$ ...(i)

As m is the root of $x^2 + px + n = 0$,

$m^2 + pm + n = 0$ ...(ii)

(ii) – (i),

$(m^2 - n^2) + p(m - n) + n - m = 0$

$(m - n)(m + n) + p(m - n) - 1(m - n) = 0$

$(m - n)(m + n + p - 1) = 0$

$\Rightarrow m + n + p - 1 = 0$ $[\because m \neq n]$

$\Rightarrow m + n + p = 1$

19. (A) $(x - 1)^2 + (x - 3)^2 + (x - 5)^2 = 0$

$x^2 - 2x + 1 + x^2 - 6x + 9 + x^2 - 10x + 25 = 0$

$3x^2 - 18x + 35 = 0$

$D = b^2 - 4ac$

$= (18)^2 - 4 \times 3 \times 35$

$= 324 - 420 < 0$

$\therefore$ The equation does not have real roots.

20. (A) $p \times a_p = q \times a_q$

$\Rightarrow p(a + (p - 1)d) = q(a + (q - 1)d)$

$\Rightarrow ap + p^2d - pd = aq + q^2d - qd$

$\Rightarrow a(p - q) + d(p^2 - q^2) - d(p - q) = 0$

$\Rightarrow a + (p + q - 1)d = 0$ $[\because p \neq q]$

$\Rightarrow a_{p+q} = 0$

21. (B) We have $p = \log x$, $q = \log(x^3)$

and $r = \log(x^5)$

$2q = 2\log(x^3) = \log x^6$

$= \log x + \log x^5$

$= p + r$

$\Rightarrow p, q$ and r are in AP.

$\therefore$ Statement is correct.

$q^2 = (\log x^3)^2 \neq (\log x)(\log x^5) \neq p \times r$

$\therefore p, q, r$ can never be in G.P.

$\therefore$ Statement II is also correct.

22. (C) x, y and z are in GP

$y^2 = xz$

$2 \log y = \log x + \log z$

$\Rightarrow \log x, \log y$ and $\log z$ are in AP

$\Rightarrow 1 + \log x, 1 + \log y$ and $1 + \log z$ are also in AP

$$\Rightarrow \frac{1}{1 + \log x}, \frac{1}{1 + \log y} \text{ and } \frac{1}{1 + \log z}$$

are in H.P.

23. (A) $Sn = n(2n + 1)$

$S_{n-1} = (n - 1)[2(n - 1) + 1]$

$= (n - 1)[2n - 1]$

$a_n = S_n - s_{n-1}$

$= 2n^2 + n - 2n^2 + 2n + n - 1$

$= 4n - 1$

24. (C) $\frac{S_p}{S_q} = \frac{p^2}{q^2}$

$$\frac{\frac{p}{2}[2a + (p - 1)d]}{\frac{q}{2}[2a + (q - 1)d]} = \frac{p^2}{q^2}$$

$$\frac{2a + pd - d}{2a + qd - d} = \frac{p}{q}$$

$(2a - d)q + pqd = (2a - d)p + pqd$

$\Rightarrow (2a - d)(p - q) = 0$

$\Rightarrow a = \frac{d}{2}$ or $d = 2a$

$[\because p \neq q]$

25. (D) Let the five consecutive terms of AP are $a - 2d, a - d, a, a + d$ and $a + 2d$.

$(a - 2d)(a - d) a (a + d)(a + 2d) = 229635$

$(a^2 - 4a^2)(a^2 - d^2)a = 229635$ ...(i)

As $a - 2d$, $a - d$ and $a + 2d$ are in G.P.,

$(a - d)^2 = (a - 2d)(a + 2d)$

$\Rightarrow a^2 + d^2 - 2ad = a^2 - 4d^2$

$\Rightarrow 5d^2 - 2ad = 0$

$\Rightarrow d(5d - 2a) = 0$

$\Rightarrow 5d - 2a = 0$ $[\because d \neq 0]$

$\Rightarrow d = \frac{2a}{5}$

From (i),

$$\left(a^2 - 4 \times \frac{4a^2}{25}\right)\left(a^2 - \frac{4a^2}{25}\right)a = 229635$$

$$\frac{9a^2 \times 21a^2 \times a}{25 \times 25} = 229635$$

$$a^5 = \frac{25 \times 25 \times 229635}{9 \times 21}$$

$= 25 \times 25 \times 1215$

$\Rightarrow \quad a = 15$

$d = 2\times\frac{15}{5} = 6$

26. (C) Required sum $= 5a$
$= 5 \times 15 = 75$

27. (C) As the roots of the given quadratic equation are equal.
D = 0

$[b^2(c^2 - a^2)]^2 - 4a^2c^2(b^2 - c^2)(a^2 - b^2) = 0$

$b^4(c^2 - a^2)^2 - 4a^2c^2(a^2b^2 - a^2c^2 - b^4 - b^2c^2)$

$b^4c^4 + b^4a^4 - 2b^4c^2a^2 - 4a^4b^2c^2 + 4a^4c^4 + 4a^2b^4c^2 - 4a^2b^2c^4 = 0$

$b^4c^4 + b^4a^4 + 4a^4c^4 + 2a^2b^4c^2 - 4a^4b^2c^2 - 4a^2b^2c^4 = 0$

$\Rightarrow (b^2c^2 + a^2b^2 - 2a^2c^2)^2 = 0$

$\Rightarrow b^2c^2 + a^2b^2 = 2a^2c^2$

$\Rightarrow \frac{2}{b^2} = \frac{1}{a^2} + \frac{1}{c^2}$

$\Rightarrow a^2, b^2$ and c^2 are in H. P.

28. (A) $x = \frac{-b^2(c^2 - a^2)}{2a^2(b^2 - c^2)}$

$= \frac{b^2(c^2 - a^2)}{a^2(c^2 - b^2)}$

29. (A) There are five odd number *i.e.* {1, 3, 5, 7, 9}
Required 4-digit number $= 5^4 = 625$

30. (B) There are 3 Vowels and 2 consonants in the letter INDIA
Number of ways Vowels can be arranged in odd places $= \frac{^3P_3}{2!}$

$= \frac{3!}{2!} = 3$

Number of ways consonant can be arrange in even places $= {^2P_2} = 2!$
Required number of ways $= 3 \times 2!$
$= 6$

31. (C) There are 5 Vowels and 3 Consonants in the word EQUATION
Required number of ways is
$= 2! \times 5! \times 3!$
$= 2 \times 120 \times 6 = 1440$

32. (D) As 2 cources are compulsory, So a student has to choose $(n - 4)$ course form $(n - 2)$ courses.
Required number of ways $= {^{n-2}C_{n-4}}$

$= \frac{(n-2)(n-3)}{2!} = \frac{(n-2)(n-3)}{2}$

33. (A) As $D_1 < D_2 < D_3$,
the required number of outcomes
$= {^6C_3}$

$= \frac{6\times5\times4}{1\times2\times3} = 20$

34. (C) We have $(1 + x)^p(1 + x)^q = (1 + x)^{p+q}$
Let $p + q = n$

$(1 + x)^n = 1 + nx + \frac{n(n-1)x^2}{2} + \frac{n(n-1)(n-2)x^3}{6} + ...$

$\frac{n(n-1)(n-2)}{6} = 35$

$n(n - 1)(n - 2) = 7 \times 6 \times 5$

$\Rightarrow n = 7$

35. (B) $7^n = (1 + 6)^n$

$= 1 + 6n + \frac{n(n-1)}{2}\times6^2 + \frac{n(n-1)(n-2)}{6}\times6^3 + ...$

$7^n - 6n$

$= 1 + 6^2\left[\frac{n(n-1)}{2} + \frac{n(n-1)(n-2)}{6}\times6 + ...\right]$

$\therefore$ When $7^n - 6n$ divided by 36 leaves a remainder as 1.

36. (D) We have $U + V = \left(8+3\sqrt{7}\right)^{20}$

and $W = \left(8-3\sqrt{7}\right)^{20}$

$\left(8+3\sqrt{7}\right)^{20} > 1$

$\Rightarrow 0 < \frac{1}{\left(8+3\sqrt{7}\right)^{20}} < 1$

$\Rightarrow 0 < \left(\frac{1}{\left(8+3\sqrt{7}\right)}\times\frac{8-3\sqrt{7}}{8-3\sqrt{7}}\right)^{20} < 1$

$\Rightarrow 0 < \left(8-3\sqrt{7}\right)^{20} < 1$

$\Rightarrow 0 < W < 1$
As V and W lies in (0,1), then $0 < V + W < 2$ ...(i)
$\therefore$ Option (D) *i.e.,* 1 satisfy equation (i)

37. (B) $(U + V)W$

$= \left(8+3\sqrt{7}\right)^{20}\left(8-3\sqrt{7}\right)^{20}$

$= \left[8^2 - \left(3\sqrt{7}\right)^2\right]^{20}$

$= 1$

38. (C) We have, $7x^2 - 6x + 1 = 0$

$\tan\alpha + \tan\beta = \frac{6}{7}$ and

$\tan\alpha \tan\beta = \frac{1}{7}$

$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$

$= \frac{\frac{6}{7}}{1-\frac{1}{7}} = 1$

$\therefore \alpha + \beta = \frac{\pi}{4}$

$2\alpha + 2\beta = \frac{\pi}{2}$ and $\alpha \neq \beta$

$\therefore$ The given triangle is right-angled.

39. (C) As $0 < x < \pi$, $0 < 5x < 5\pi$,
$\text{Cot}2x\ \text{Cot}3x = 1$
$\Rightarrow \text{Cot}2x = \tan3x$
$\Rightarrow 2x + 3x =$
$\frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \frac{9\pi}{2}$
Hence, there are five required solutions.

40. (A) $\text{Cos}^{100}x - \sin^{100}x = 1$
$\text{Cos}^{100}x = 1 + \sin^{100}x$
As $\sin^{100}x \in [0, 1]$
and $\cos^{100}x \in [0, 1]$,
$\sin^{100}x = 0$ or
$\cos^{100}x = 1$
$\Rightarrow x = n\pi$, where n is an integer.

41. (C) $\text{Sin}12°\ \text{Sin}48° =$
$\frac{-1}{2}(\cos 60° - \cos 36°)$

$= \frac{-1}{2}\left[\frac{1}{2} - \left(\frac{\sqrt{5}+1}{4}\right)\right]$

$= \frac{-1}{2}\left[\frac{2-\sqrt{5}-1}{4}\right]$

$= \frac{\sqrt{5}-1}{8}$

42. (D) $\frac{\cos 17° - \sin 17°}{\cos 17° + \sin 17°}$

$= \frac{1 - \tan 17°}{1 + \tan 17°}$

$= \tan(45° - 17°)$
$= \tan 28°$
$= \cot 62°$

43. (C) Let $x = 22.5$
$\tan 2x = 1$

$\frac{2\tan x}{1 - \tan^2 x} = 1$

$\Rightarrow \tan^2 x + 2\tan x - 1 = 0$

$\Rightarrow \tan x = \frac{-2 \pm \sqrt{4+4}}{2}$

$= \sqrt{2} - 1$

$\therefore$ tan 22.5 is irrational

$\cot 22.5 = \frac{1}{\tan 22.5}$

$= \frac{1}{\left(\sqrt{2}-1\right)}\times\frac{\left(\sqrt{2}+1\right)}{\left(\sqrt{2}+1\right)}$

$= \sqrt{2} + 1$

$\therefore$ cot 22.5 is also irrational

tan 22.5 – cot 22.5

$= \sqrt{2}-1-\sqrt{2}-1$

$= -2$

Which is rational.

44. (B) $\dfrac{x}{\cos\theta} = \dfrac{y}{\cos\left(\dfrac{2\pi}{3}-\theta\right)}$

$= \dfrac{z}{\cos\left(\dfrac{2\pi}{3}+\theta\right)}$

$= \dfrac{x+y+z}{\cos\theta+\cos\left(\dfrac{2\pi}{3}-\theta\right)+\cos\left(\dfrac{2\pi}{3}+\theta\right)}$ ...(i)

Now,

$= \cos\theta+\cos\left(\dfrac{2\pi}{3}-\theta\right)+\cos\left(\dfrac{2\pi}{3}+\theta\right)$

$= \cos\theta+2\cos\left(\dfrac{2\pi}{3}\right)\cos(\theta)$

$= \cos\theta\left[1+2\cos\left(\pi-\dfrac{\pi}{3}\right)\right]$

$= \cos\theta\left[1-2\cos\dfrac{\pi}{3}\right] = 0$

$\Rightarrow x+y+z = 0$ [from (i)]

45. (D) $p\tan(\theta-30°) = q\tan(\theta+120°)$

$\dfrac{p}{q} = \dfrac{\tan(\theta+120°)}{\tan(\theta-30°)}$

$\dfrac{p+q}{p-q}$

$= \dfrac{\tan(\theta+120°)+\tan(\theta-30°)}{\tan(\theta+120°)-\tan(\theta-30°)}$

Let $\theta + 120° = A$ and $\theta - 30° = B$

$\dfrac{p+q}{p-q} = \dfrac{\dfrac{\sin A}{\cos A}+\dfrac{\sin B}{\cos B}}{\dfrac{\sin A}{\cos A}-\dfrac{\sin B}{\cos B}}$

$= \dfrac{\sin(A+B)}{\sin(A-B)}$

$= \dfrac{\sin(2\theta+90)}{\sin(150)}$

$= 2\cos 2\theta$

46. (B) We have, $a\cos x + b\sin x + c$

Maximum Value of $a\cos + b\sin x$ is $\sqrt{a^2+b^2}$

$\therefore$ Maximum Value of $a\cos x + b\sin x + c$ is $\sqrt{a^2+b^2}+c$

47. (A) We have $\dfrac{a}{\cos A} = \dfrac{b}{\cos B} = \dfrac{c}{\cos C}$

But we know that

$\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

$\Rightarrow$ Tan A = Tan B = Tan C

$\Rightarrow$ A = B = C

Δ ABC is an equilateral triangle

Area of equilateral triangle

$= \dfrac{\sqrt{3}}{4}\times c^2 = 9\sqrt{3}$sq.cm

48. (B) $\angle C = 180 - (75+45) = 60°$

$\dfrac{a}{\sin 75} = \dfrac{b}{\sin 45} = \dfrac{c}{\sin 60}$

$\dfrac{a\times 2\sqrt{2}}{\sqrt{3}+1} = \dfrac{b\sqrt{2}}{1} = \dfrac{2c}{\sqrt{3}}$

$\Rightarrow$ $a = \dfrac{(\sqrt{3}+1)}{\sqrt{6}}c$

and $b = \dfrac{2c}{\sqrt{6}}$

$2a - b = \dfrac{2c}{\sqrt{6}}(\sqrt{3}+1-1)$

$= \dfrac{2\sqrt{3}c}{\sqrt{6}}$

$= \sqrt{2}c$

49. (B) tan A + tan B + tan C = K

As $A + B + C = \pi$

$\Rightarrow$ tan A + tan B + tan C

= tan A · tan B · tan C

$\Rightarrow$ tan A tan B tan C $= k$

cot A cot B cot C $= \dfrac{1}{k}$

50. (C) $4\sin^{-1}x + \cos^{-1}x = \pi$

$4\left(\dfrac{\pi}{2}-\cos^{-1}x\right)+\cos^{-1}x = \pi$

$2\pi - 3\cos^{-1}x = \pi$

$\cos^{-1}x = \dfrac{\pi}{3}$

$x = \cos\dfrac{\pi}{3}$

$\sin^{-1}x + 4\cos^{-1}x$

$\Rightarrow$ $\dfrac{\pi}{2}+3\cos^{-1}x$

$\left[\therefore \cos^{-1}x+\sin^{-1}x = \dfrac{\pi}{2}\right]$

$\Rightarrow$ $\dfrac{\pi}{2}+\dfrac{3\pi}{3} = \dfrac{3\pi}{2}$

51. (B) $\cot^2(\sec^{-1}2) + \tan^2(\text{cosec}^{-1}3)$

$\text{cosec}^2(\sec^{-1}2)-1+\sec^2(\text{cosec}^{-1}3)-1$

$= \text{cosec}^2\left(\text{cosec}^{-1}\dfrac{2}{\sqrt{3}}\right)$

$+\sec^2\left(\sec^{-1}\dfrac{3}{2\sqrt{2}}\right)-2$

$= \dfrac{4}{3}+\dfrac{9}{8}-2$

$= \dfrac{32+27+48}{24}$

$= \dfrac{11}{24}$

52. (D) ZY matrix has the order 3 × 3 (ZY) X is defined and has order 3 × 3

$\therefore$ Statement 1 is correct.

XZ has order 3 × 2

Y(XZ) has order 2 × 2

$\therefore$ Y(XZ) is defined and has square matrix of order 2.

$\therefore$ Statement II is also correct.

YZ has order 2 × 2, but X(YZ) is not defined.

$\therefore$Statement III is also correct.

53. (C) We have, $P = \begin{bmatrix} 0 & c & -b \\ -c & 0 & a \\ b & -a & 0 \end{bmatrix}$

and $Q = \begin{bmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{bmatrix}$

$PQ = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ = a null matrix

$QP = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \neq I$

$\therefore$ PQ = QP

54. (C) If AB is a null matrix, then B is a null matrix, if A is a non-singular matrix.

55. (B) $Z = \dfrac{1}{3}\begin{vmatrix} i & 2i & 1 \\ 2i & 3i & 2 \\ 3 & 1 & 3 \end{vmatrix} = x + iy$

$\Rightarrow \dfrac{1}{3}[i(9i-2)-2i(6i-6)+1(2i-9i)]$

$= x + iy$

$\Rightarrow \dfrac{1}{3}[-9-2i+12+12i-7i] = x + iy$

$\dfrac{1}{3}[3+3i] = x + iy$

$\Rightarrow$ $x = 1$ and $y = 1$

$z = 1 + i$

$|z| = \sqrt{2}$

56. (A) We have, $\begin{vmatrix} 1+\omega & 1+\omega^2 & \omega+\omega^2 \\ 1 & \omega & \omega^2 \\ \dfrac{1}{\omega} & \dfrac{1}{\omega^2} & 1 \end{vmatrix}$

$\begin{vmatrix} -\omega^2 & -\omega & -1 \\ 1 & \omega & \omega^2 \\ \omega^2 & \omega & 1 \end{vmatrix}$

$[\because 1+\omega+\omega^2 = 0$ and $\omega^3 = 1]$

$= 0$

57. (A) $D_n = \begin{vmatrix} n & 20 & 30 \\ n^2 & 40 & 50 \\ n^3 & 60 & 70 \end{vmatrix}$

$\sum_{n=1}^{4} D_n$

$= \begin{vmatrix} 1+2+3+4 & 20 & 30 \\ 1^2+2^2+3^2+4^2 & 40 & 50 \\ 1^3+2^3+3^3+4^3 & 60 & 70 \end{vmatrix}$

$= \begin{vmatrix} 10 & 20 & 30 \\ 30 & 40 & 50 \\ 100 & 60 & 70 \end{vmatrix}$

$= 1000\begin{vmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 10 & 6 & 7 \end{vmatrix}$

$= 1000\,[1(28-30) - 2(21-50) + 3(18-40)]$

$= 1000[-2 + 58 - 66]$

$= -10000$

58. (B) The determinant of a skew-symmetric matrix is always zero

$\therefore$ $|P| = 0$, as P is a skew symmetric matrix

59. (D) We have, $A = \begin{bmatrix} 3 & -3 & 4 \\ 0 & -3 & 4 \\ 0 & -4 & 1 \end{bmatrix}$

$A(\text{adj } A) = |A| I_3$

$= \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix} I_3$

$= \begin{bmatrix} 3 & 1 & 4 \\ 2 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix} I_3$

$= 1\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

60. (C) $|A| = 1$

$\text{Adj } A = \begin{bmatrix} 1 & -2 & -2 \\ -1 & 3 & 3 \\ 0 & -4 & -3 \end{bmatrix}$

$= \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|}\text{adj}A = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}$

61. (C) Maximum number of intersecting points in 4 straight lines $= {}^4C_2 = 6$

Maximum number of intersecting points between straight line and *a* circle $= {}^4C_1 \times 2 = 8$

$\therefore$ Required number of possible points $= 6 + 8 = 14$

62. (D) Slope of diagonal, which is along line

$x - 2y = 1$ is $\frac{1}{r}$ (let m_1)

Slope of diagonal, which is along

$4x + 2y = 3$ is $\frac{-4}{2}$

$= -2$ (let m_2)

As $m_1 \times m_2 = \frac{1}{r} \times (-2)$

$= -1$

the two diagonals are perpendicular to each other.

$\therefore$ The given quadrilateral ABCD must be a rhombus.

63. (B) The diagonals of parallelogram bisect each other.

Let O be the intersecting point of diagonal PR and QS.

Coordinate of O is $\left(\frac{2+10}{2}, \frac{4+14}{2}\right)$

or $\left(\frac{8+x}{2}, \frac{12+y}{2}\right)$

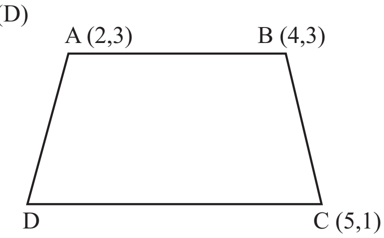
$\Rightarrow \quad \frac{8+x}{2} = 6$

and $\quad \frac{12+y}{2} = 9$

$\Rightarrow \quad x = 4$ and $y = 6$

$\therefore \quad x + y = 4 + 6 = 10$

64. (D)



Let coordinate of D be (x, y)

Slope of AB = Slope of DC

$\frac{3-3}{4-2} = \frac{1-y}{5-x}$

$\Rightarrow \quad y = 1$

$AD = BC$

$\sqrt{(3-y)^2 + (2-x)^2} = \sqrt{(3-1)^2 + (4-5)^2}$

$4 + (2-x)^2 = 4 + 1$

$\Rightarrow \quad 2 - x = \pm 1$

$\Rightarrow \quad 2 - x = 1$ or $2 - x = -1$

$\Rightarrow \quad x = 1$ or $x = 3$

If $x = 1$, then ABCD will become parallelogram.

$\therefore \quad x = 3$

Hence, the point of vertex D is (3, 1)

65. (C) Equation of AC is

$y - 1 = \frac{(1-3)(x-5)}{(5-2)}$

$\Rightarrow \quad y - 1 = \frac{-2(x-5)}{3}$

$\Rightarrow \quad 2x = 13 - 3y \quad$ ...(i)

Equation of BD is

$y - 3 = \frac{(3-1)(x-4)}{(4-3)}$

$\Rightarrow \quad y - 3 = 2x - 8$

$\Rightarrow \quad y - 3 = 13 - 3y - 8$

$\Rightarrow \quad 4y = 8$

$\Rightarrow \quad y = 2$

Substitute y = 2 in (i), we get

$2x = 13 - 6 = 7$

$x = \frac{7}{2}$

$\therefore$ The required point is $\left(\frac{7}{2}, 2\right)$.

66. (B) We have,

$4x^2 + 9y^2 = 1$

$\frac{x^2}{\frac{1}{4}} + \frac{y^2}{\frac{1}{9}} = 1$

Here, $\quad a^2 = \frac{1}{4}$ and $b^2 = \frac{1}{9}$

Sum of the distances of any point on the ellipse from its foci $= 2a$

$\therefore \quad PQ + PR = 2 \times \frac{1}{2} = 1$

67. (A) We have,

$(x^2 - 4x + 3) + (y^2 - 6y + 8) = 0$

$(x - 3)(x - 1) + (y - 2)(y - 4) = 0$

$\therefore$ There are 2 possibility of end points of diameter.

i.e., (1, 4) and (3, 2) or (1, 2) and (3, 4)

$\therefore$ Statements I and II are correct.

68. (B) $y^2 = 4kx$

Coordinate of A is (0, 0).

Slope of PA $= \frac{4k - 0}{4k - 0} = 1$

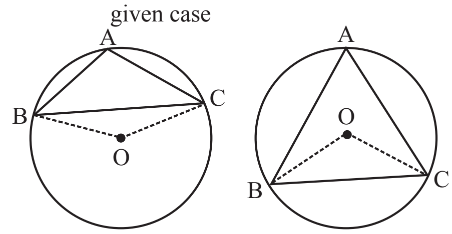
Slope of QA $= \frac{-4k - 0}{4k - 0} = -1$

As slope of PA × Slope of QA = –1,

$PA \perp QA$

$\Rightarrow \quad \angle PAQ = 90°$

69. (C) There are 2 triangle possible for the given case



Case 1 Case 2

Given equation of circle is

$x^2 + y^2 = 100$

Coordinate of center 0 is (0, 0) and radius *i.e.*, BO = OC = 10 unit

$$BC = \sqrt{(8-6)^2 + (6+8)^2}$$

$$= 10\sqrt{2}$$

ΔBOC is an right triangle with hypotenuse BC

$$\angle BOC = 90° \text{ or } \frac{\pi}{2}$$

For case 1,

$$\text{Reflex angle BOC} = 2\pi - \frac{\pi}{2} = \frac{3\pi}{2}$$

$$\angle BAC = \frac{11}{2} \times \text{Relfex angle BOC}$$

$$= \frac{1}{2} \times \frac{3\pi}{2} = \frac{3\pi}{4}$$

For case 2,

$$\angle BAC = \frac{1}{2} \times \angle BOC$$

$$= \frac{1}{2} \times \frac{\pi}{2} = \frac{\pi}{4}$$

$\therefore \angle BAC$ is either $\frac{\pi}{4}$ or $\frac{3\pi}{4}$.

70. (D) Coordinate of A can't be determine due to insufficient data.

71. (D)

$$|x-3| = \begin{cases} x-3 & \text{if } x \geq 3 \\ 3-x & \text{if } x < 3 \end{cases}$$

$$\frac{|x-3|}{x-3} = \begin{cases} 1 & \text{if } x \geq 3 \\ -1 & \text{if } x < 3 \end{cases}$$

As LHL $\neq$ RHL at $x = 3$, limit does not exist.

72. (B) We have lines (sec θ – tan θ)

$$\theta \to \frac{\pi}{2}$$

$$= \lim_{\theta \to \frac{\pi}{2}} \left(\frac{1 - \sin\theta}{\cos\theta}\right) \quad \left(\frac{0}{0}\right)$$

Using L's Hospital Rule

$$= \lim_{\theta \to \frac{\pi}{2}} \left(\frac{-\cos\theta}{-\sin\theta}\right)$$

$$= \lim_{\theta \to \frac{\pi}{2}} \cot\theta = 0$$

73. (C)

$$f(x) = [x]^2 - [x^2]$$

$$f(0.999) = [0.999]^2 - [(0.999)^2]$$

$$= 1^2 - [0.998001]$$

$$= 1 - 0 = 1$$

$$f(1.001) = [1.001]^2 - [(1.001)^2]$$

$$= 1^2 - [1.002001]$$

$$= 1 - 1 = 0$$

$$\therefore f(0.999) + f(1.001) = 1 + 0 = 1$$

74. (D) $f(x)$ is not continuous at $x = 1$ as $f(0.999) \neq f(1.001)$

$$\lim_{x \to 0^-} f(x) = [x]^2 - [x^2]$$

$$= (-1)^2 - 0 = 1$$

$$\lim_{x \to 0^+} f(x) = [x]^2 - [x^2]$$

$$= 0 - 0 = 0$$

$\therefore f(x)$ is also not continuous at $x = 0$

75. (D)

$$f(0) = -1, f'(0) = 1$$

$$h(x) = f[2f(x) + 2]$$

and $g(x) = [h(x)]^2$

$$h'(x) = f'[2\{f(x)+2\}]\{2f'(x)\}$$

$$h'(0) = f'[2f(0) + 2] \times 2f'(0)$$

$$= 2f'(0) \times f'(0)$$

$$= 2 \times 1 \times 1 = 2$$

76. (A)

$$g(x) = [h(x)]^2$$

$$g'(x) = 2h(x) \times h'(x)$$

$$g'(0) = 2h(0) \times h'(0)$$

$$= 2 \times [f\{2f(0) + 2\}] \times 2$$

$$= 4 \times [f(0)]$$

$$= 4 \times (-1) = -4$$

77. (B)

$$f(x) = \cos 2x + x$$

$$f'(x) = -2\sin 2x + 1$$

$$f''(x) = -2 \times 2\cos 2x$$

$$= -4\cos 2x$$

Put $f'(x) = 0$

$$-2\sin 2x + 1 = 0$$

$$\text{sim } 2x = \frac{1}{2}$$

$$\Rightarrow \quad 2x = \frac{\pi}{6}, \frac{5\pi}{6}$$

as $x \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

$$\Rightarrow \quad x = \frac{\pi}{12}, \frac{5\pi}{12}$$

$$f''\left(\frac{\pi}{12}\right) = -4\cos\frac{\pi}{6} < 0$$

$$f''\left(\frac{5\pi}{12}\right) = -4\cos\frac{5\pi}{6} > 0$$

$\therefore f(x)$ is maxima at $x = \frac{\pi}{12}$

$$f\left(\frac{\pi}{12}\right) = \cos\frac{\pi}{6} + \frac{\pi}{12}$$

$$= \frac{\sqrt{3}}{2} + \frac{\pi}{12}$$

78. (*) $f(x)$ is minima at $x = \frac{5x}{12}$

$$f\left(\frac{5\pi}{12}\right) = \cos\frac{5\pi}{6} + \frac{5\pi}{12}$$

$$= \frac{1}{2} + \frac{5\pi}{12}$$

79. (B) Let $I = 2\int \frac{x^2-1}{\sqrt{x^2+1}} dx$

$$= 2\int\left(\frac{x^2-1}{\sqrt{x^2+1}} - \frac{2}{\sqrt{x^2+1}}\right)dx$$

$$= 2\int\sqrt{x^2+1}dx - 4\int\frac{1}{\sqrt{x^2+1}}dx$$

$$= 2\left[\frac{x}{2}\sqrt{x^2+1} + \frac{1}{2}\log\left|x + \sqrt{x^2+1}\right|\right]$$

$$-4\log\left|x + \sqrt{x^2+1}\right|$$

$$= x\sqrt{x^2+1} - 3\log\left|x + \sqrt{x^2+1}\right|$$

Let $v(x) = x$

and $v(x) = \sqrt{x^2+1}$

$$|u^2(x) - v^2(x)| = |x^2 - x^2 - 1|$$

$$= 1$$

80. (A)

$$u(x)v(x) = x\sqrt{x^2+1}$$

$$= \sqrt{x^4 + x^2}$$

81. (D)

$$f(x) = |x^2 - x - 2|$$

$$= |(x-2)(x+1)|$$

$$f(x) = |(x-2)(x+1)|$$

$$= \begin{cases} (2-x)(-x-1) & x < -1 \\ (2-x)(x+1) & 1 \leq x < 2 \\ (x-2)(x+1) & x > 2 \end{cases}$$

So, $\int_0^2 f(x)dx = \int_0^2 (2-x)(x+1)dx$

$$= \int_0^2 (2x - x^2 + 2 - x)dx$$

$$= \int_0^2 (x + 2 - x^2)dx$$

$$= \left[\frac{x^2}{2} + 2x - \frac{x^3}{3}\right]_0^2$$

$$= \frac{4}{2} + 4 - \frac{8}{3}$$

$$= 6 - \frac{8}{3} = \frac{10}{3}$$

82. (B) $\int_1^3 f(x)$

$$= \int_1^2 (x + 2 - x^2)dx$$

$$+ \int_2^3 (x^2 - x - 2)dx$$

$= \left[\frac{x^2}{2} + 2x - \frac{x^3}{3}\right]_1^2 + \left[\frac{x^3}{3} - \frac{x^2}{2} - 2x\right]_2^3$

$= \left(\frac{4}{2} + 4 - \frac{8}{3}\right) - \left(\frac{1}{2} + 2 - \frac{1}{3}\right) + \left(\frac{27}{3} - \frac{9}{2} - 6\right) - \left(\frac{8}{3} - 6\right)$

$= 6 - \frac{8}{3} - \frac{1}{2} - 2 + \frac{1}{3} + 3 - \frac{9}{2} - \frac{8}{3} + 6$

$= 13 - 5 - 5 = 3$

83. (C) $f(t) = \log\left(t + \sqrt{1+t^2}\right)$

$f(-t) = \log\left(-t + \sqrt{1+t^2}\right)$

$= \log\left(\frac{1}{-t + \sqrt{1+t^2}}\right)^{-1}$

$= -\log\left[\frac{1}{(-t+\sqrt{1+t^2})} \times \frac{(t+\sqrt{1+t^2})}{(t+\sqrt{1+t^2})}\right]$

$= -\log\left(\frac{t+\sqrt{1+t^2}}{-t^2+1+t^2}\right)$

$= -\log(t+\sqrt{1+t^2}) = -f(t)$

$\therefore$ $f(t)$ is an odd function

$g(t) = \tan(f(t))$

$g(-t) = \tan(f(-t))$

$= \tan(-f(t))$

$= -\tan(f(t))$

$= -g(t)$

$\therefore$ $g(t)$ is also an odd function

Hence, both statements are correct.

84. (B) $\int_{-\pi}^{\pi} g(t)dt = 0$

as $g(t)$ is an odd function

85. (C) $\int_0^{\frac{\pi}{2}} \frac{dx}{g(x)} = \int_0^{\frac{\pi}{2}} \frac{1}{\sin x + \cos x + 1} dx$

$= \int_0^{\frac{\pi}{2}} \frac{1+\tan^2\frac{x}{2}}{2\tan\frac{x}{2} + 1 - \tan^2\frac{x}{2} + 1 + \tan^2\frac{x}{2}} dx$

Let $\tan\frac{x}{2} = t$

then $\sec^2\frac{x}{2}dx = 2dt$

When $x \to 0$, then $t \to 0$ and when $x \to \frac{\pi}{2}$, $t \to 1$

$= \int_0^1 \frac{2}{2t+2} dt$

$= [\log|t+1|]_0^1$

$= \log 2 - \log 1 = \log 2$

86. (C) $I = \int_0^{\frac{\pi}{2}} \frac{\sin x}{\sin x + \cos x + 1} dx$

$= \int_0^{\frac{\pi}{2}} \frac{2\tan\frac{x}{2}}{2\tan\frac{x}{2} + 1 - \tan^2\frac{x}{2} + 1 + \tan^2\frac{x}{2}} dx$

$= \int_0^{\frac{\pi}{2}} \frac{\tan\frac{x}{2}}{\tan\frac{x}{2} + 1} dx$

Let $\tan\frac{x}{2} = t$

$\Rightarrow$ $\sec^2\frac{x}{2}dx = 2dt$

$\Rightarrow$ $dx = \frac{2}{1+t^2}dt$

$I = \int_0^1 \frac{2t}{(1+t^2)(1+t)} dt$

$\frac{2t}{(1+t^2)(1+t)} = \frac{At+B}{1+t^2} + \frac{C}{1+t}$

$2t = At^2 + At + B + Bt + C + Ct^2$

$\Rightarrow A + C = 0, A + B = 2, B + C = 0$

$\Rightarrow A = 1, B = 1, C = -1$

$I = \int_0^1 \left(\frac{t+1}{t^2+1} - \frac{1}{1+t}\right) dt$

$= \int_0^1 \left(\frac{1}{2} \times \frac{2t}{t^2+1} + \frac{1}{t^2+1} - \frac{1}{1+t}\right) dt$

$= \left[\frac{1}{2}\log|t^2+1| + \tan^{-1}t - \log|1+t|\right]_0^1$

$= \frac{1}{2}\log 2 + \frac{\pi}{4} - \log 2$

$= \frac{\pi}{4} - \frac{1}{2}\log 2$

87. (B) $y^2 = kx$

and the line $x = k$

Area $= 2\int_0^k \sqrt{k}\sqrt{x}dx$

$= 2\sqrt{k}\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^k$

$= \frac{4}{3}\sqrt{k}(k)^{\frac{3}{2}}$

$= \frac{4}{3}k^2$

$\Rightarrow$ $\frac{4}{3}k^2 = \frac{4}{3}$

$\left[\because \text{ given area} = \frac{4}{3}\text{sq. units}\right]$

$\Rightarrow$ $k = 1$ $[\because k > 0]$

88. (A) Equation of parabola is $y^2 = x$

Compare it with $y^2 = 4ax$

$\Rightarrow$ $4a = 1$ and $a = \frac{1}{4}$

Required area $= 2\int_0^{\frac{1}{4}} \sqrt{x}dx$

$= 2\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^{\frac{1}{4}}$

$= \frac{4}{3} \times \left(\frac{1}{4}\right)^{\frac{3}{2}}$

$= \frac{1}{6}$sq. units

89. (A) We have,

$ydx + (x - y^3)dy = 0$

$\Rightarrow$ $\frac{dy}{dx} = \frac{y}{y^3 - x}$

This is a differential equation whose order and degree are 1 and 1 respectively.

90. (D) $\frac{dx}{dy} + \frac{1}{y}x = y^2$

This is a linear differential equation

I.F. $= e^{\int P dy}$

$= e^{\int \frac{1}{y}dy}$

$= e^{\log y} = y$

$x.y = \int y.y^2 dy + c$

$xy = \frac{y^4}{4} + c$

$4xy - y^4 = c$

91. (B) $(2\hat{i} + 6\hat{j} + 27\hat{k}) \times (\hat{i} + \alpha\hat{j} + \beta\hat{k}) = 0$

$\begin{vmatrix} i & j & k \\ 2 & 6 & 27 \\ 1 & \alpha & \beta \end{vmatrix} = 0$

$i(6\beta - 27\alpha) - j(2\beta - 27) + k(2\alpha - 6) = 0$

$\Rightarrow$ $6\beta - 27\alpha = 0,$

$2\beta - 27 = 0,$

$2\alpha - 6 = 0$

$\Rightarrow$ $\beta = \frac{27}{2}$

and $\alpha = 3$

$\therefore$ $3\alpha + 2\beta = 2 \times 3 + 2 \times \frac{27}{2}$

$= 33$

92. (B) $|\vec{a} \times \vec{b}| + \sqrt{3}|\vec{a}.\vec{b}|$

$= |\vec{a}||\vec{b}|\sin\theta + \sqrt{3}|\vec{a}||\vec{b}|\cos\theta$

$= |\vec{a}||\vec{b}|[\sin\theta + \sqrt{3}\cos\theta]$

$= 2|\vec{a}||\vec{b}|\left[\frac{1}{2}\sin\theta + \frac{\sqrt{3}}{2}\cos\theta\right]$

$= 2|\vec{a}||\vec{b}|\left[\sin\left(\theta+\frac{\pi}{3}\right)\right]$

The quantity $|\vec{a}\times\vec{b}|+\sqrt{3}|\vec{a}.\vec{b}|$ is maximum, when

$\theta+\frac{\pi}{3} = \frac{\pi}{2}$

[∵ Max. Value of $\sin\theta = 1$]

$\Rightarrow \quad \theta = \frac{\pi}{2}-\frac{\pi}{3}$

$= \frac{\pi}{6} = 30°$

93. (A) $|\vec{a}| = |\vec{b}| = 1$

$(\vec{a}+2\vec{b}).(5\vec{a}-4\vec{b}) = 0$

$[\because (\vec{a}+25)$ is perpendicular to $5\vec{a}-4\vec{b}]$

$5|\vec{a}|^2-8|\vec{b}|^2+6|\vec{a}||\vec{b}|\cos\theta = 0$

$-3+6\cos\theta = 0$

$\cos\theta = \frac{1}{2}$

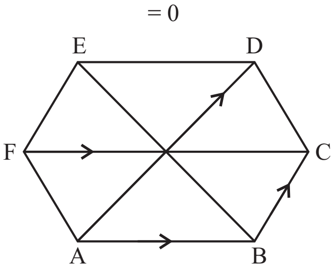
$\Rightarrow \quad \theta = \frac{\pi}{3}$

$\cos\theta+\cos 2\theta = \cos\frac{\pi}{3}+\cos\frac{2\pi}{3}$

$= \cos\frac{\pi}{3}+\cos\left(\pi-\frac{\pi}{3}\right)$

$= 0$

94. (A)



Since ABCDEF is a regular hexagon,

$\overrightarrow{AD} = 2\overrightarrow{BC}$

and $\overrightarrow{FC} = 2\overrightarrow{AB}$

$\Rightarrow \quad \overrightarrow{CF} = -2\overrightarrow{AB}$

Here, $m = 2$ and $n = -2$

$mn = 2\times(-2) = -4$

95. (A) Let θ be the angle formed by vectors $\vec{a}, \vec{b}$ and $\vec{c}$, when taken pair wise.

Let $\vec{c} = x\hat{i}+y\hat{j}+z\hat{k}$

$|\vec{a}| = \sqrt{2}$

and $|\vec{b}| = \sqrt{2}$

$\vec{a}.\vec{c} = \sqrt{2}|\vec{c}|\cos\theta$

$x+y = \sqrt{2}|\vec{c}|\cos\theta$ ...(i)

$\vec{b}.\vec{c} = \sqrt{2}|\vec{c}|\cos\theta$

$\Rightarrow \quad y+z = \sqrt{2}|\vec{c}|\cos\theta$ ...(ii)

Compare (i) and (ii), we get $x = z$

$\vec{a}.\vec{b} = (\sqrt{2})(\sqrt{2})\cos\theta$

$\frac{1}{2} = \cos\theta$

$\Rightarrow \quad \theta = \frac{\pi}{3}$

From equation (i), we get

$x^2+y^2 = 2|\vec{c}|^2\times\left(\frac{1}{2}\right)^2$

$\Rightarrow \quad x^2+y^2 = 2(x^2+y^2+z^2)\times\frac{1}{4}$

$\Rightarrow 2x^2+2y^2 = x^2+y^2+z^2$

$\Rightarrow \quad 2y^2 = y^2$

$\Rightarrow \quad y = 0$

$\therefore$ $\vec{c}$ can be equal to $\hat{i}+\hat{k}$

$\therefore$ Only statement I is correct.

96. (B) We have,

$2x^2+2y^2+2z^2+3x+3y+3z-6 = 0$

$x^2+\frac{3}{2}x+y^2+\frac{3}{2}y+z^2+\frac{3}{2}z = 3$

$\left(x+\frac{3}{4}\right)^2+\left(y+\frac{3}{4}\right)^2+\left(z+\frac{3}{4}\right)^2$

$= 3+3\times\frac{9}{16}$

$\left(x+\frac{3}{4}\right)^2+\left(y+\frac{3}{4}\right)^2+\left(z+\frac{3}{4}\right)^2$

$= \frac{75}{16}$

$\therefore$ Centre of sphere is $\left(\frac{-3}{4},\frac{-3}{4},\frac{-3}{4}\right)$

and radius is $\sqrt{\frac{75}{16}}$ or $\frac{5\sqrt{3}}{4}$

$\therefore$ Diameter of the sphere

$= 2\times\frac{5\sqrt{3}}{4}$

$= \frac{5\sqrt{3}}{2}$

97. (D) From option (A),

$2x+2y+2z-3 = 2(x+y+z)-3$

$= 2\left(\frac{-3}{4}-\frac{3}{4}-\frac{3}{4}\right)-3$

$= \frac{-9}{2}-3 \neq 0$

From option (B),

$4x+4y+4z-3 = 4(x+y+z)-3$

$= 4\left(\frac{-3}{4}-\frac{3}{4}-\frac{3}{4}\right)-3$

$= -9-3 \neq 0$

From option (C),

$4x+8y+8z-15$

$= -3-6-6-15 \neq 0$

From option (D),

$4x+8y+8z+15$

$= -3-6-6+15 = 0$

$\therefore$ Option (D) is correct answer.

98. (B) Let a, b and c be the direction ratio of intersection of two planes

$\Rightarrow a+b+c = 0$

and $2a+3b-4c = 0$

$\frac{a}{-7} = \frac{-b}{-6} = \frac{c}{1}$

$\therefore$ Direction ratio of the required line are <–7, 6, 1>.

99. (A) Direction cosine of S is

$<\frac{-7}{\sqrt{49+36+1}}, \frac{6}{\sqrt{49+36+1}}, \frac{1}{\sqrt{49+36+1}}>$

$= <\frac{-7}{\sqrt{86}}, \frac{6}{\sqrt{86}}, \frac{1}{\sqrt{86}}>$

Here $\quad l = \frac{-7}{\sqrt{86}}$,

$m = \frac{6}{\sqrt{86}}$,

$n = \frac{1}{\sqrt{86}}$

$\therefore 43(l^2-m^2-n^2)$

$= 43\left[\frac{49}{86}-\frac{36}{86}-\frac{1}{86}\right]$

$= 43\times\frac{12}{86} = 6$

100. (C) L : $x+y+z+4 = 0$

$= 2x-y-z+8$

Let a, b, c be the direction ratios

$a+b+c = 0$ and $2a-b-c = 0$

$\Rightarrow \quad \frac{a}{0} = \frac{-b}{-3} = \frac{c}{-3}$

$\therefore$ Direction ratios are <0, 1, –1>

101. (B) Put $y = 0$

we get $x+z+4 = 0$

and $2x-z+8 = 0$

$\Rightarrow \quad x = -4, z = 0$

$\therefore$ L passes through (–4, 0, 0)

Equation of L is

$\frac{x+4}{0} = \frac{y-0}{1}$

$= \frac{z-0}{-1} = \lambda$ (let)

$x = -4, y = \lambda, z = -\lambda$

The point $(-4, \lambda, -\lambda)$ passes through P,

$-4+2\lambda-3\lambda+1 = 0$

$-3-\lambda = 0$

$\Rightarrow \quad \lambda = -3$

$\therefore$ The intersecting point of L and P is (4, –3, 3).

102. (D) We have, $x - 3y + 4 = 0$ and
$2x - 7y + 8 = 0$

$\Rightarrow \quad y = \frac{2}{7} + \frac{8}{7}$

and $\quad x = 3y - 4$

$\Rightarrow \quad b_{yx} = \frac{2}{7}$ and $b_{xy} = 3$

$\therefore \quad b_{xy} + 7b_{yx} = 3 + 2 = 5$

103. (B) Mean = $\frac{1+4+9+16+...n^2}{n}$

$\Rightarrow \frac{(n+1)(2n+1)}{6} = 130$

$\Rightarrow 2n^2 + 3n + 1 = 780$
$\Rightarrow 2n^2 + 3n - 779 = 0$
$\Rightarrow 2n^2 + 41n - 38n - 779 = 0$
$\Rightarrow n(2n + 41) - 19(2n + 41) = 0$
$\Rightarrow \quad n = 19$

104. (B) Mean of first 10 natural number

$= \frac{1+2+3+4+...+10}{10} = 5.5$

M.D. ($\bar{x}$)

$= \frac{|1-5.5|+|2-5.5|+|3-5.5| +|4-5.5|+|5-5.5| +...|10-5.5|}{10}$

$= \frac{4.5+3.5+2.5+1.5+0.5+0.5 +1.5+2.5+3.5+4.5}{10}$

$= 2.5$

105. (B) $\sum_{i=1}^{9} x_i^2 = 855$

$\Rightarrow x_1^2 + x_2^2 + ... + x_9^2 = 855$

$\sigma^2 = \frac{\Sigma x_i^2}{n} - (\bar{x})^2$

$\sigma^2 + M^2 = \frac{\Sigma x_i^2}{n}$

$= \frac{855}{9} = 95$

106. (D) $\bar{x} = \frac{x_1 + x_2 + ...x_n}{n}$

$\Rightarrow x_1 + x_2 + ... + x_{n-1} = n\bar{x} - x_n$

New Mean = $\frac{x_1 + x_2 + ... + x_{n-1} + k}{n}$

$= \frac{n\bar{x} - x_n + k}{n}$

107. (D) Mean = $np = 6$
Variance = $npq = 2$

$\Rightarrow \quad q = \frac{2}{6} = \frac{1}{3}$

$p = 1 - \frac{1}{3} = \frac{2}{3}$

$n \times \frac{2}{3} = 6$

$\Rightarrow \quad n = 9$

108. (C) Total number of outcomes of selecting 3 distinct numbers = ${}^{10}C_3$

$= \frac{10 \times 9 \times 8}{1 \times 2 \times 3}$

$= 120$

Favourable Outcomes are (1, 2, 3), (2, 3, 4), (3, 4, 5), (4, 5, 6), (5, 6, 7), (6, 7, 8), (7, 8, 9) (8, 9, 10) *i.e.,* 8 in number.

Required probability = $\frac{8}{120} = \frac{1}{15}$

109. (C) $3P(B) = 4P(A)$
and $3P(C) = 2P(B)$
P(A) + P(B) + P(C) = 1 as A, B and C are mutually exclusive and exhaustive event.

$\frac{3}{4}P(B) + P(B) + \frac{2}{3}P(B) = 1$

$\frac{9P(B) + 12P(B) + 8P(B)}{12} = 1$

$P(B) = \frac{12}{29}$

$P(A) = \frac{3}{4} \times \frac{12}{29} = \frac{9}{29}$

110. (C) $P(6) = \frac{1}{6}$

$P(4 \text{ or } 5) = 1 - P(6)$

$= 1 - \frac{1}{6} = \frac{5}{6}$

111. (A) Total number of outcomes of drawing 3 balls = $12 \times 12 \times 12 = 12^3$
Favourable outcomes = $2^3 + 4^3 + 6^3$

Req. Probability = $\frac{2^3 + 4^3 + 6^3}{12^3}$

$= \frac{288}{12 \times 12 \times 12}$

$= \frac{1}{6}$

112. (A) $P(A) = \frac{5}{6}$,

$P(B) = \frac{4}{5}$

$P(C) = \frac{3}{4}$

$P(A\bar{B}C) = P(A) \times P(B) \times P(C)$

$= \frac{5}{6} \times \frac{1}{5} \times \frac{3}{4} = \frac{1}{8}$

113. (D) Total number of outcomes

$= \frac{7!}{3!}$

$= 7 \times 6 \times 5 \times 4$
$= 840$

Consonants and vowel occur alternatively iff letter O comes at even position.

$\therefore$ Number of ways the letter 'O' comes at even places = $\frac{3!}{3!} = 1$

$\therefore$ Number of ways the consonants comes at odd places = 4!
Number of favourable outcomes
$= 4! \times 1 = 24$

$\therefore$ Required probability

$= \frac{24}{840}$

$= \frac{1}{35}$

114. (A) Let E be a event that a natural number x is chosen from first 100 natural number such that $x^2 + x > 50$

$\bar{E} = \{1, 2, 3, 4, 5, 6, 7\}$

$P(E) = 1 - P(\bar{E})$

$= 1 - \frac{7}{100}$

$= \frac{93}{100}$

115. (C) $P(\text{Head}) = P(\text{Tail}) = \frac{1}{2}$

Required Probability
= P(HH) + P(THH) + P(TTHH) + P(TTTHH) + P(TTTTHH)

$= \frac{1}{2^2} + \frac{1}{2} \times \frac{1}{2^2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2^2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2^2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2^2}$

$= \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^4} + \frac{1}{2^5} + \frac{1}{2^6}$

$= \frac{1}{2^2} \frac{\left(1 - \frac{1}{2^5}\right)}{1 - \frac{1}{2}}$

$= \frac{1}{2}\left(\frac{2^5 - 1}{2^5}\right)$

$= \frac{2^5 - 1}{2^6}$

$= \frac{31}{64}$

116. (B) Let E_1 be the event that white ball is transferred from urn A to B and E_2 is the event that black balls is transferred from urn A to B.

$P(E_1) = \frac{2}{4}$

$= \frac{1}{2} = P(E_2)$

Let A be the event that the ball drawn in white

$$P\left(\frac{A}{E_1}\right) = \frac{4}{6}$$

$$P\left(\frac{A}{E_2}\right) = \frac{3}{6}$$

$$P(A) = P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right)$$

$$= \frac{1}{2}\times\frac{4}{6}+\frac{1}{2}\times\frac{3}{6}$$

$$= \frac{7}{12}$$

117. (D) $P(A) = P\left(\frac{A}{B}\right) = 0.25$

$\Rightarrow \frac{P(A\cap B)}{P(B)} = 0.25$ ...(i)

$$P\left(\frac{B}{A}\right) = \frac{P(A\cap B)}{P(A)}$$

$= 0.5$

$\Rightarrow P(A\cap B) = 0.5 \times 0.25$

$= 0.125$

From (i), $P(B) = \frac{0.125}{0.25}$

$= \frac{1}{2} = 0.5$

$P(A)P(B) = 0.25 \times 0.5$

$= 0.125$

$= P(A\cap B)$

∴ A and B are independent

∴ Statement I is correct.

$P(A^C \cup B^C) = P(A\cap B)^C$

$= 1 - P(A\cap B)$

$= 1 - 0.125$

$= 0.875$

∴ Statement II is also correct.

$P(A^C \cap B^C) = P(A\cup B)^C$

$= 1 - P(A\cup B)$

$= 1 - P(A) - P(B) + P(A\cap B)$

$= 1 - 0.25 - 0.5 + 0.125$

$= 0.375$

Statement III is also correct.

118. (D) There are only 7 cases when sum of dice are either 9 or 10 *i.e.,* (3, 6), (4, 5), (5, 4), (6, 3), (4, 6), (5, 5) and (6, 4)

Required Probability

$= 1 - P(\text{sum is either 9 or 10})$

$= 1 - \frac{7}{36}$

$= \frac{29}{36}$

119. (A) P(worker has disease) *i.e.*

$$p = 20\% = \frac{1}{5}$$

$$q = 1-\frac{1}{5} = \frac{4}{5}$$

Here, $n = 6$

$P(X \geq 4)$

$= P(X = 4) + P(X = 5) + P(X = 6)$

$= {}^6C_4 p^4 q^2 + {}^6C_5 p^5 q + {}^6C_6 p^6$

$= 15\times\frac{1}{5^4}+\frac{16}{5^2}+6\times\frac{1}{5^5}\times\frac{4}{5}+\frac{1}{5^6}$

$= \frac{240+24+1}{15625}$

$= \frac{265}{15625}$

$= \frac{53}{3125}$

120. (D) Total number of outcomes

$= 6 \times 5 \times 4 = 120$

There are 3 cases for the given condition

Ace		
	Ace	
		Ace

∴ Number of ways in each case

$= 5 \times 4 = 20$

Number of favourable outcomes

$= 20 \times 3$

$= 60$

∴ Required Probability $= \frac{60}{120} = \frac{1}{2}$

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