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Chapter 1

Sets, Relations and Functions

1. Introduction

In Mathematical language all living and non-living things in universe are known as objects. A set is well defined class or collection of objects. Every item in the set is called an element of the set. A set is usually represented by the capital letter and its elements are denoted by small letters.

Some of the examples of the sets are:

(i) The set of all even numbers.

(ii) The set of all books written about travel to Chile.

(iii) {1, 3, 9, 12}

(iv) {red, orange, yellow, green, blue, indigo, purple}

2. Representation of Sets

There are different set notations used for the representation of sets. They differ in the way in which the elements are listed. The three set notations used for representing sets are :

(i) Roster form

(ii) Set builder form

(i) Roaster Form : In this method a set is described by listing elements, separated by commas, within braces {}. The set of vowels of English alphabet may be described as $\{a, e, i, o, u\}$.]

In a roster form, the order of the elements of the set does not matter, for example, the set of the first five odd numbers can also be defined as {1, 5, 9, 3, 7}

If there is an endless list of elements in a set, then they are defined using a series of dots at the end of the last element. For example, infinite sets are represented as, X = {1, 2, 3, 4, 5 ...}, where X is the set of natural numbers.

The elements in the roaster form should not be repeated

(ii) Set-builder Form or Rule method : The set builder form uses a vertical bar in its representation, with a text describing the character of the elements of the set.

In this method, a set is described by a characterizing property (let $P(x)$) of its elements (let x). The symbol '|' or ':' is used to separate the elements and properties. In such a case, the set is described by $\{x$:properties of $x\}$ or $\{x \mid$ properties of $x\}$, which is read as 'the set of all x such that $P(x)$ holds'. The symbol '|' or ':' is read as 'such that'.

The set A = {0, 1, 4, 9, 16 ...} can be written as $A = \{x^2 | x \in Z\}$.

3. Sets Symbols

Set symbols are used to define the elements of a given set. The following table shows some of these symbols and their meaning :

Symbol	Meaning
$\Rightarrow$	Implies
$\in$	Belongs to
$A \subset B$	A is a subset of B
$\Leftrightarrow$	Implies and is implied by
$\notin$	Does not belong to
s.t.(: or |)	Such that
$\forall$	For every
$\exists$	There exists
iff	If and only if
&	And
$a \mid b$	a is a divisor of b
N	Set of natural numbers
I or Z	Set of integers
R	Set of real numbers
C	Set of complex numbers
Q	Set of rational numbers

4. Types of Sets

Sets are classified into different types. The following table shows some of different types of sets :

(i) Null set or Empty set : The set which contains no element is called the null set or empty set or the void set. It is denoted by the symbol ϕ or { }. For example, the set of all even prime number greater than 2 is a empty set.

(ii) Singleton Sets : A set that has only one element is called a singleton set or unit set. For example, the set of all even prime number is a singleton set as 2 is the only even prime number.

(iii) Finite Sets : A set that has a finite or countable number of elements is called a finite set. Example, Let set $B = \{k \mid k$ is a prime number less than 20}, which can also be written as B = {2, 3, 5, 7, 11, 13, 17, 19}. As the set B contains 8 elements, it is a finite set.

(iv) Infinite Sets : A set that has an infinite number of elements is called an infinite set. Example: Set $C = \{k \mid k$ is a even number}, which can be written as C = {2, 4, 6, 8, 10, …}.

As there are infinite even numbers, the set C is a infinite set

(v) Equal Sets : If two non-empty sets have the same elements, then the two sets are known as equal. For example: let set A = {1, 2, 3} and set B = {3, 1, 2}. Here, set A and set B contains the same elements, so they are equal sets. This can be represented as A = B.

(vi) Equivalent Sets : Two non-empty sets are said to be equivalent sets if they have the same number of elements, though the elements are different. Example: let set A = {1, 2, 3, 4} and set B = {a, b, c, d}. Here, set A and set B are equivalent sets since n(A) = n(B)

(vii) Disjoint Sets : Two sets are disjoint sets if there are no common elements in both sets. Example: let set A = {1, 2, 3, 4, 5}and set B = {6, 7, 8, 9, 10}. Here, no element is common in set A and set B. Therefore, A and B are disjoint sets.

(viii) Subset and Superset : For two sets A and B, if every element in set A is present in set B, then set A is a subset of set B($A \subseteq B$) and B is the superset of set A($B \supseteq A$). For example: let set A = {1, 2, 3, 4, 5} and set B = {1, 2, 3, 4, 5, 6, 7, 8, 9}. Since all the elements in set A are present in set B, then $A \subseteq B$ or $B \supseteq A$.

Note

- Null set is the subset of each set
- Each set is the subset of itself
- If A has n elements, then the number of subsets of set A is 2^n.

(ix) Proper subsets : If each element of A is in set B but set B has at least one element which is not in A and A ≠ B, then set A is known as proper subset of set B. If A is a proper subset of B, then it is written as $A \subset B$ and read as A is a proper subset of B. For example: let set A = {1, 2} and set B = {1, 2, 3, 8, 9}. Since all the elements in set A are present in set B, then $A \subset B$

Note

- The null set ϕ is subset of every set and every set is subset of itself, *i.e.*, $\phi \subset A$ and $A \subseteq A$ for every set A. They are called improper subsets of A. Thus every non-empty sets has two improper subsets. It should be noted that ϕ has only one subset ϕ which is improper. All other subsets of A are called its proper subsets. Thus, if $A \subset B$, $A \neq B$, $A \neq \phi$ then A is said to be proper subset of B.
- No. of proper subset of any set of n elements = $2^n - 1$

(x) Universal Set : A universal set is the collection of all the elements in regard to a particular subject. The universal set is denoted by the letter 'U'. For example: Let set U = {The list of all road transport vehicles}. Here, a set of cars is a subset for this universal set, the set of cycles, trains are all subsets of this universal set.

(xi) Power Set : Power set is the set of collection of all the subsets that a set could contain. For example: Let set A = {1, 2, 3}. Power set of A is {{ϕ}, {1}, {2}, {3}, {1, 2}, {2, 3}, {1, 3}, {1, 2, 3}}.

Examples

Q. 1. Which of the following is the empty set ?
(A) {x : x is a real number and $x^2 - 1 = 0$}
(B) {x : x is a real number and $x^2 + 1 = 0$}
(C) {x : x is a real number and $x^2 - 9 = 0$}
(D) {x : x is a real number and $x^2 - x - 2 = 0$}

Sol. : (B) Since $x^2 + 1 = 0$, gives $x^2 = -1 \Rightarrow x = \pm i$

$\therefore$ x is not real but it is given that x is real

Hence, no value of x is possible.

Q. 2. The number of proper subsets of the set {1, 2, 3} is :

(A) 8 (B) 7
(C) 6 (D) 5

Sol. : (B) Number of proper subsets of the set {1, 2, 3} = $2^3 - 1$ = 7 *i.e.*, {ϕ},{1}, {2}, {3}, {1, 2}, {2, 3}, {1, 3}

Q. 3. Let S = {0, 1, 5, 4, 7}. Then the total number of subsets of S is :

(A) 64 (B) 32
(C) 40 (D) 20

Sol. : (B) Number of subsets of the set {0, 1, 5, 4, 7} = 2^5
= 32

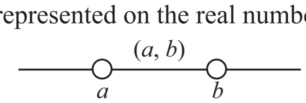
5. Intervals as a Subset of Real Numbers

- Interval Notation is a method of representing a subset of real numbers by those numbers that bound them. We can use this notation to describe inequalities
- There are different types of notations of intervals that are classified based on the endpoints of intervals. They are :

(i) Open intervals
(ii) Closed intervals
(iii) Half-open intervals

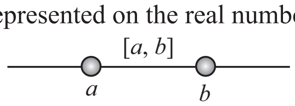
(i) Open Intervals : The set of all the real numbers between a and b *i.e.*, {x : $a < x < b$} is called an open interval. It is denoted by (a, b) or [a, b]. Here, open intervals contain all the points between a and b belonging to (a, b), but a, b themselves do not belong to this interval.

This can be represented on the real number line as:



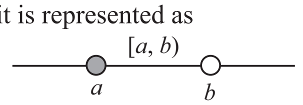
(ii) Closed Intervals : The set of all the real numbers between a and b in which a and b are also included *i.e.*, {x : $a \le x \le b$} is called an closed interval. It is denoted by [a, b].

This can be represented on the real number line as :

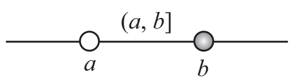


(iii) Half open Intervals : The set of all the real numbers between a and b in which either a or b is included and the other is excluded.

The set $\{x : a \le x < b\}$ is an open interval from a to b, which contain all the points between a and b including a but excluding b. It is denoted by $[a, b)$ or $[a, b[$. Graphically, it is represented as



The set $\{x : a < x \le b\}$ is an open interval from a to b, which contain all the points between a and b including b but excluding a. It is denoted by $(a, b]$ or $]a, b]$. Graphically, it is represented as



(iv) Degenerate Interval : A set consisting of a single real number or an interval of the form a to a, *i.e.* $[a, a]$ is called a degenerate interval.

(v) Bounded and Unbounded Intervals : An interval is considered bounded if both endpoints are real numbers. An interval is unbounded if both endpoints are not real numbers. For example, the interval (1, 5) is considered bounded and $(-\infty, \infty)$ is considered unbounded.

Intervals that are bounded at only one end are called half-bounded.

An interval is said to be left-bounded if there is some real number that is smaller than all its elements and is called a right-bounded if there is some real number that is larger than all its elements.

Example

Q. 1. Let $A = [x : x \in R, |x| < 1]$, $B = [x : x \in R, |x - 1| \ge 1]$ and $A \cup B = R - D$, then the set D is :

(A) $[x : 1 < x \le 2]$ (B) $[x : x \in R, 1 \le x < 2]$

(C) $[x : x \in R, 0 \le x < 2]$ (D) None of these

Sol. : (B) $A = [x : x \in R, -1 < x < 1]$

$B = [x : x \in R : x - 1 \le -1 \text{ or } x - 1 \ge 1]$

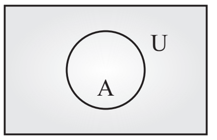
$= [x : x \in R : x \le 0 \text{ or } x \ge 2]$

$\therefore \quad A \cup B = R - [1, 2)$

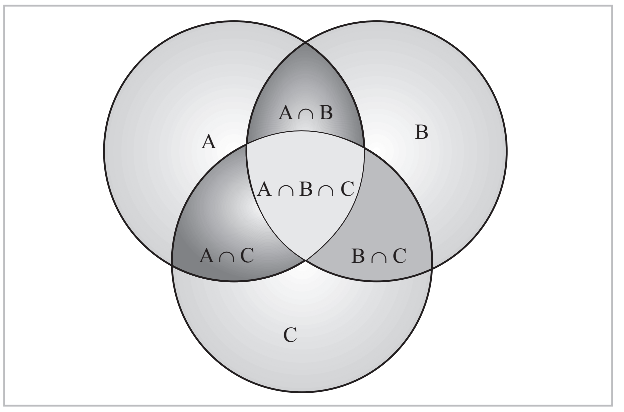
Hence, $D = [x : x \in R, 1 \le x < 2]$.

6. Venn-Euler Diagrams

- A Venn diagram is a diagram used to represent the sets, relation between the sets and operation performed on them, in a pictorial way.
- A Venn diagram is also called a set diagram or a logic diagram showing different set operations such as the intersection of sets, union of sets and difference of sets.
- It is also used to depict subsets of a set. The universal set (U) is usually represented by a closed rectangle, consisting of all the sets. The sets and subsets are shown by using circles or oval shapes.



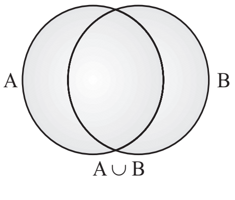
- Two disjoints sets are represented by two non-intersecting circles.
- The Venn-diagram of three set is shown in the diagram below :



7. Operations on Sets

7.1 Union of Two or more Sets

- Union of two or more sets is the set containing all the elements of the given sets. Union of sets can be written using the symbol "$\cup$".
- Let A and B be two non-empty sets. The union of A and B is the set of all elements which are in set A or in B. The union of A and B by $A \cup B$, which is usually read as "A union B". For example, Let's consider an example, let set $A = \{1, 3, 5\}$ and set $B = \{1, 2, 4\}$ then $A \cup B = \{1, 2, 3, 4, 5\}$
- Symbolically, the union of a set can be represented as $A \cup B = \{x : x \in A \text{ or } x \in B\}$
- Graphically, it is represented as shown in the figure.

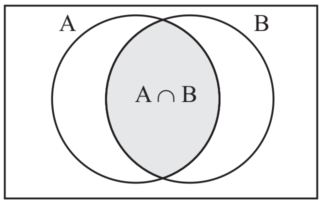


7.2 Intersection of Two or more Sets

- Intersection of two or more sets is the set containing all the common elements of the given sets. Intersection of sets can be written using the symbol "$\cap$".
- Let A and B be two non-empty sets. The intersection of A and B is the set of all common elements which are in both set A and in B. The intersection of A and B by

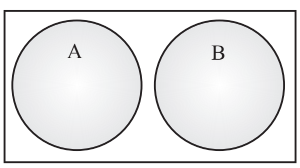
$A \cap B$, which is usually read as "A intersection B". For example, Let's consider an example, let set A = {1, 3, 5, 6} and set B = {1, 2, 4, 6} then $A \cup B$ = {1, 6}

- Symbolically, the intersection of a set can be represented as $A \cap B = \{x : x \in A \text{ and } x \in B\}$
- Graphically, it is represented as shown in the figure.



7.3 Disjoint of Two or more Sets

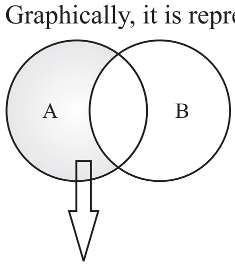
- Disjoint of two or more sets is the set which does not have common elements in the given sets.
- Let A and B be two non-empty sets. The sets A and B are said to be disjoint if $A \cap B = \phi$ *i.e.*, if A and B have no common element. Let's consider an example, let set A = {3, 5, 6} and set B = { 2, 4, 7} then $A \cap B$ = { } or ϕ.
- Graphically, it is represented as shown in the figure.



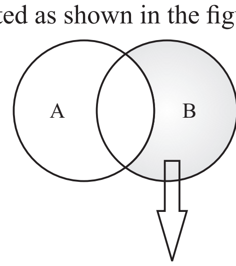
7.4 Difference of Two Sets

- Let A and B be two non-empty sets. The difference of A and B is the set of all those elements of A which do not belong to B. It is denoted as A – B. If the set contains all those elements of B but do not belong to A, then the difference of two sets is denoted as B – A. For example, Let's consider an example, let set A = {1, 3, 5, 6} and set B = {1, 2, 4, 6} then A – B = {3, 5} and B – A = {2, 4}.
- Symbolically, the difference of a set can be represented as

  $A - B = \{x : x \in A \text{ and } x \notin B\}$ or $B - A = \{x : x \notin A \text{ and } x \in B\}$
- Graphically, it is represented as shown in the figure.



A – B



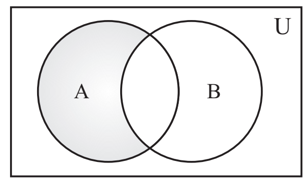
B – A

7.5 Symmetric Difference of Two Sets

- Let A and B be two non-empty sets. The symmetric difference of A and B is the set of all those elements which belongs either to A or to B but not to the their common part.
- Symmetric Difference of sets can be written using the symbol "Δ".

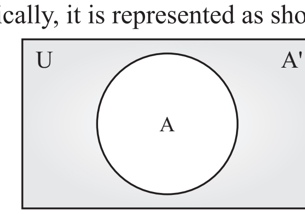
  For example, Let's consider an example, let set A = {1, 3, 5, 6} and set B = {1, 2, 4, 6} then $A \Delta B$ = {2, 3,4, 5}
- Symbolically, the symmetric difference of a set can be represented as

  $A \Delta B = (A - B) \cup (B - A)$ or $A \Delta B = (A \cup B) - (A \cap B)$
- Graphically, it is represented as shown in the figure.



7.6 Complement of a Set

- The complement of a non-empty set A is the set of all those elements of universal set which do not belong to A. It is denoted as A'. For example, Let's consider an example, let set A = {1, 3, 5, 6} and universal set U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} then A' = {2, 4, 7, 8, 9, 10}
- Symbolically, the complement of a set A can be represented as $A' = \{x : x \in U \text{ and } x \in A\}$
- Graphically, it is represented as shown in the figure.



8. Cardinal Number of a Finite and Infinite Set

The number of distinct elements in a finite set A is called cardinal number and it is denoted by $n(A)$. And if it is not finite set, then it is called infinite set.

e.g., If A = {–3, –1, 8, 10, 13, 17}, then $n(A) = 6$

9. Some Important Results on Number of Elements in Sets

If A, B and C are finite sets and U be the finite universal set, then

(i) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

(ii) $n(A \cup B) = n(A) + n(B) \Leftrightarrow$ A, B are disjoint non-empty sets.

(iii) $n(A - B) = n(A) - n(A \cap B)$ *i.e.*, $n(A - B) + n(A \cap B) = n(A)$

(iv) $n(A \Delta B) = n(A) + n(B) - 2n(A \cap B)$

(v) $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

(vi) n elements in exactly two of the sets A, B, C $= n(A \cap B) + n(B \cap C) + n(C \cap A) - 3n(A \cap B \cap C)$

(vii) n of elements in exactly one of the sets A, B, C $= n(C) - 2n(A \cap B) - 2n(B \cap C) - 2n(C \cap A) + 3n(A \cap B \cap C)$

(viii) $n(A' \cup B') = n(A \cap B)' = n(U) - n(A \cap B)$

(ix) $n(A' \cap B') = n(A \cup B)' = n(U) - n(A \cup B)$

10. Laws of Algebra of Sets

- **Idempotent laws :** For any set A, we have

  (i) $A \cup A = A$ (ii) $A \cap A = A$

- **Identity laws :** For any set A, we have

  (i) $A \cup \phi = A$ (ii) $A \cap U = A$

  i.e., ϕ and U are identity elements for union and intersection respectively.

- **Commutative laws :** For any two sets A and B, we have

  (i) $A \cup B = B \cup A$ (ii) $A \cap B = B \cap A$

  (iii) $A \Delta B = B \Delta A$

  i.e., union, intersection and symmetric difference of two sets are commutative.

  (iv) $A - B \neq B - A$ *i.e.*, difference of two sets is not commutative

- **Associative laws :** If A, B and C are any three sets, then

  (i) $(A \cup B) \cup C = A \cup (B \cup C)$

  (ii) $A \cap (B \cap C) = (A \cap B) \cap C$

  (iii) $(A \Delta B) \Delta C = A \Delta (B \Delta C)$

  i.e., union, intersection and symmetric difference of two sets are associative.

  (iv) $(A - B) - C \neq A - (B - C)$ *i.e.*, difference of two sets is not associative.

- **Distributive law :** If A, B and C are any three sets, then

  (i) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

  (ii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

  i.e., union and intersection are distributive over intersection and union respectively.

  (iii) $A - (B \cap C) = (A - B) \cup (A - C)$

  (iv) $A - (B \cup C) = (A - B) \cap (A - C)$

  (v) $A \cap (B - C) = (A \cap B) - (A \cap C)$

  (vi) $A \cap (B \Delta C) = (A \cap B) \Delta (A \cap C)$

- **De-Morgan's law :** If the complements of the intersection of A and B will be equal to the union of A' and B'.

  (i) $(A \cup B)' = A' \cap B'$

  (ii) $(A \cap B)' = A' \cup B'$

- If A and B are any two sets, then

  (i) $A - B = A \cap B'$

  (ii) $B - A = B \cap A'$

  (iii) $A - B = A \Leftrightarrow A \cap B = \phi$

  (iv) $(A - B) \cup B = A \cup B$

  (v) $(A - B) \cap B = \phi$

  (vi) $A \subseteq B \Leftrightarrow B' \subseteq A'$

  (vii) $(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$

Examples

Q. 1. If $A = \{2, 3, 4, 8, 10\}$, $B = \{3, 4, 5, 10, 12\}$, $C = \{4, 5, 6, 12, 14\}$ then $(A \cap B) \cup (A \cap C)$ is equal to

(A) $\{3, 4, 10\}$ (B) $\{2, 8, 10\}$

(C) $\{4, 5, 6\}$ (D) $\{3, 5, 14\}$

Sol. : (A) $A \cap B = \{3, 4, 10\}$

$A \cap C = \{4\}$

$(A \cap B) \cup (A \cap C) = \{3, 4, 10\}$

Q. 2. If $A = [-3, 7]$ and $B = [2, 9]$, then which of the following is not true ?

(A) $A \cap B = [2, 7]$

(B) $(A \cup B)' = (-\infty, -3) \cup (9, \infty)$

(C) $A - B = [-3, 2]$

(D) $A - B' = [2, 7]$

Sol. : (C) Given $A = [-3, 7], B = [2, 9]$

$A \cup B = [-3, 9]$

$A \cap B = [2, 7]$

$(A \cup B)' = (-\infty, -3,) \cup (9, \infty)$

$A - B = [-3, 2)$

$B' = (-\infty, 2) \cup (9, \infty)$

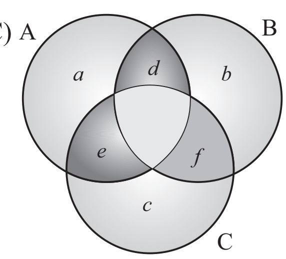
$A - B' = [2, 7].$

Q. 3. Let A, B and C be finite sets such that $A \cap B \cap C = \phi$ and each one of the sets $A \Delta B$, $B \Delta C$ and $C \Delta A$ has 100 elements. The number of elements in $A \cup B \cup C$ is :

(A) 250 (B) 200

(C) 150 (D) 300

Sol. : (C)



$A \Delta B = 100$

$\Rightarrow a + e + b + f = 100$ ...(i)

$B \Delta C = 100$

$\Rightarrow b + d + c + e = 100$ ...(ii)

$C \Delta A = 100$

$\Rightarrow c + f + a + d = 100$ ...(iii)

Adding all the three equations

$2(a + b + c + d + e + f) = 300$

$a + b + c + d + e + f = 150$

$\Rightarrow A \cup B \cup C = 150.$

Q. 4. In a town of 10,000 families it was found that 40% family buy newspaper A, 20% buy newspaper B and 10% families buy newspaper C, 5% families buy A and B, 3% buy B and C and 4% buy A and C. If 2% families buy all the three newspapers, then number of families which buy A only is :

(A) 3100 (B) 3300
(C) 2900 (D) 1400

Sol. : (B) n (U) = 10,000

n(A) = 40% of 10,000 = 4000

n(B) = 20 % of 10,000 = 2000

n(C) = 10% of 10,000 = 1000

$n(A \cap B)$ = 5% of 10,000 = 500

$n(B \cap C)$ = 3% of 10,000 = 300

$n(A \cap C)$ = 4% of 10,000 = 400

$n(A \cap B \cap C \cap)$ = 2% of 10,000 = 200

$n(\text{only A}) = n(A) - [n(A \cap B) + n(A \cap C)] + n(A \cap B \cap C)$

$= 4000 - (500 + 400) + 200$

$= 3300.$

Q. 5. If A and B are two sets, then $A \cup B = A \cap B$ iff :

(A) $A \subseteq B$ (B) $B \subseteq A$
(C) $A = B$ (D) None of these

Sol. : (C) If $A \subseteq B$, then $A \cup B = B$ and $A \cap B = A$

If $B \subseteq A$, then $A \cup B = A$ and $A \cap B = B$

If $A = B$, then $A \cup B = A \cap B = A = B$

Q. 6. If A and B are sets, then $A \cap (B - A)$ is :

(A) ϕ (B) A
(C) B (D) None of these

Sol. : (A) $A \cap (B - A) = A \cap (B \cap A') = A \cap (A' \cap B)$

$= (A \cap A') \cap B$

$= \phi \cap B$

$= \phi.$

11. Ordered Pairs

- A pair of elements written in a particular order is called an ordered pair. It is written by listing its two elements in a particular order, separated by a comma and enclosing the pair in brackets.
- In the ordered pair (a, b), a is called the first component or the first coordinate and b is called the second component or the second coordinate.

12. Cartesian Product of Sets

- The Cartesian products of sets mean the product of two non-empty sets in an ordered way. In other words, the collection of all ordered pairs obtained by the product of two non-empty sets. An ordered pair means that two elements are taken from each set.
- Let A and B be any two non-empty sets. The set of all ordered pairs (a, b) such that $a \in A$ and $b \in B$ is called the Cartesian product of the sets A and B and is denoted by $A \times B$. Thus, $A \times B = [(a, b) : a \in A \text{ and } b \in B]$
- For example, Let $A = \{a, b, c\}$ and $B = \{p, q\}$, then $A \times B = \{(a, p), (a, q), (b, p), (b, q), (c, p), (c, q)\}$. Also $B \times A = \{(p, a), (p, b), (p, c), (q, a), (q, b), (q, c)\}$
- Number of elements in A×B is the product of the number of elements in set A and B *i.e.*, $n(A \times B) = n(A) \times n(B)$

Note

- If $A = \phi$ or $B = \phi$, then we define $A \times B = \phi$.
- If one of A and B is an infinite set and the other is a non-empty set, then the Cartesian product $A \times B$ is an infinite set.

12.1 Properties of Cartesian Product

- The Cartesian product does not follows commutative law, *i.e.*, $A \times B \neq B \times A$
- If A and B are any two non-empty sets, then $A \times B = B \times A \Leftrightarrow A = B$
- $A \times B = \varphi$, if either $A = \varphi$ or $B = \varphi$.
- The Cartesian product follows associative law *i.e.*, $(A \times B) \times C = A \times (B \times C)$.
- The Cartesian product follows Distributive law *i.e.*, For any three sets A, B, C;
  (i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$
  (ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$
  (iii) $A \times (B - C) = (A \times B) - (A \times C)$
- If $A \subseteq B$, then $A \times A \subseteq (A \times B) \cap (B \times A)$
- If $A \subseteq B$, then $A \times C \subseteq B \times C$ for any set C.
- If $A \subseteq B$ and $C \subseteq D$, then $A \times C \subseteq B \times D$
- For any sets A, B, C, D ; $(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$
- For any three sets A, B, C
  (i) $A \times (B' \cup C')' = (A \times B) \cap (A \times C)$
  (ii) $A \times (B' \cap C')' = (A \times B) \cup (A \times C)$

Examples

Q. 1. Let A = {1, 3, 5}, B = {4, 6, 8} and C = {5, 6, 7, 9}. Find $A \times (B \cap C)$

(A) {(1, 4) (3, 6) (5, 6)} (B) {(1, 5) (3, 6) (5, 7)}
(C) {(1, 6) (3, 6) (5, 6)} (D) {(1, 4) (3, 5) (5, 6)}

Sol. : (C) **Given:**

A = {1, 3, 5}, B = {4, 6, 8} and C = {5, 6, 7, 9}

$B \cap C = \{6\}$

$A \times (B \cap C) = \{1, 3, 5\} \times \{6\}$

$\Rightarrow$ {(1, 6) (3, 6) (5, 6)}

Q. 2. If A and B are sets such that $n(A \times B) = 6$ and $A \times B$ contains (1, 2), (2, 1) and (3, 2), then find the sets A, B.

(A) A = {1, 2}, B = {1, 2, 3}

(B) A = {1, 3}, B = {1, 2, 3}

(C) A = {1, 2, 3}, B = {1, 2}

(D) None of the above

Sol. : (C) $A \times B = \{(1, 2), (2, 1), (3, 2)\}$

$\Rightarrow$ $1, 2, 3 \in A$ and $1, 2 \in B$

$n(A) = 3, n(B) = 2$

Hence, set A = {1, 2, 3} and set B = {1, 2}

13. Relations

- Let A and B be two non-empty sets, then every subset of $A \times B$ defines a relation from A to B and every relation from A to B is a subset of $A \times B$.
- Let $R \subseteq A \times B$ and $(a, b) \in R$. Then we say that "a is R related to b" or "a is related to b with respect to R" or "a and b have relation R". It is usually denoted by $a \, R \, b$

13.1 Total Number of Relations

- The total number of relations that can be defined from a set A to a set B is the number of possible subsets of $A \times B$. If $n(A) = p$ and $n(B) = q$, then $n(A \times B) = pq$ and the total number of relations is 2^{pq}.

13.2 Domain and Range of Relation

- Let R be a relation from a set A to a set B. Then the set of all first components or coordinates of the ordered pairs belonging to R is called the domain of R, while the set of all second components or coordinates of the ordered pairs in R is called the range of R. The whole set B is called the codomain of the relation R. Note that range $\subset$ codomain.
- Thus, Domain (R) = $\{a : (a, b) \in R\}$ and Range (R) = $\{b : (a, b) \in R\}$.

14. Types of Relations

(i) Void /Empty Relation: A relation R in a set A is called empty (void) relation if no element of A is related to any element of A *i.e.*, $R = \phi \subset A \times A$.

Let's take an example, let A is the set of all the students in a Girl's school. Here, the relation R "is a brother of" is a void or empty relation.

(ii) Universal Relation : A relation R in a set A is called universal relation if every element of A is related to every element of A *i.e.*, $R = A \times A$.

Let's take an example, the difference between the heights of any two living beings is less than 6 metres is an universal relation.

(iii) Identity Relation : Let A be a set. Then the relation $I_A = \{(x, x) : x \in A\}$ on A is called the identity relation on A.

In other words, a relation I_A on A is called the identity relation if every element of A is related to itself only.

For example, Let set A = {1, 2, 3}, R = {(1, 1), (2, 2), (3, 3)} is the identity relation on A.

(iv) Inverse Relation : Let A, B be two sets and let R be a relation from a set A to a set B. Then the inverse of R, denoted by R^{-1}, is a relation from B to A and is defined by

Clearly $(a, b) \in R \Leftrightarrow (b, a) \in R^{-1}$. Also, Dom (R) = Range (R^{-1}) and Range (R) = Dom (R^{-1})

For example, Let A = $\{a, b, c\}$, B = {1, 2, 3, 4} and R = $\{(a, 1), (a, 2), (b, 3), (c, 3)\}$.

Then, (i) $R^{-1} = \{(1, a), (2, a), (3, b), (3, c)\}$

(ii) Dom (R) = $\{a, b, c\}$ = Range (R^{-1})

(iii) Range (R) = {1, 2, 3} = Dom (R^{-1})

15. Classification of Relations

(i) Reflexive Relation : A relation R on a set A is said to be reflexive if every element of A is related to itself. Thus, R is reflexive $\Leftrightarrow (x, x) \in R$ for all $x \in A$.

Let's take some examples.

(A) The relation "is a factor of" in the set of rational numbers is reflexive because every ration at number is the factor of itself.

(B) The relation "is a predecessor of" in the set of natural numbers is not reflexive as no number is a predecessor of itself.

(C) Let A = {1, 2, 3} and R = {(1, 1); (1, 3)}. Then R is not reflexive since $3 \in A$ but $(3, 3) \notin R$.

Note

- Every identity relation is always a reflexive relation but every reflexive relation need not be an identity relation.
- The universal relation on a non-void set A is reflexive relation.

(ii) Symmetric Relation : A relation R on a set A is said to be a symmetric relation *iff* $(x, y) \in R \Rightarrow (y, x) \in R$ for all $x, y \in A$ *i.e.*, $xRy \Rightarrow yRx$ for all $x, y \in A$. R is symmetric *iff* $R^{-1} = R$.

Let's take some examples.

(A) The relation "is perpendicular to" in the set of straight lines is symmetry because if the line l is perpendicular to the line m, then m is also perpendicular to l.

(B) The relation "is more than" in the set of numbers is not symmetry because if a is greater than b, then b is not greater than a

(C) Let A = {1, 2, 3} and R = {(1, 2); (2, 1);(1, 3);(3, 1)}. Then R is symmetry.

Note

- Every identity relation is always a symmetry relation but every symmetry relation need not be an identity relation.
- The universal relation on a non-void set A is symmetry relation.
- A reflexive relation on a set A is not necessarily symmetric.

(iii) Anti-symmetric Relation : Let A be any set. A relation R on set A is said to be an anti-symmetric relation *iff* $(x, y) \in R$ and $(y, x) \in R \Rightarrow x = y$ for all $x, y \in A$. or it can be defined as relation R is anti symmetric if $x \neq y$ then x may be related to y or x may be related to x, but never both.

For example, the relation R = {(5,5),(1,1),(7,7),(3,3),(8,8)} is anti symmetric defined on set A = {1,3,5,7,8}

Note

- If a relation is not symmetric, that does not mean it is anti-symmetric.

Symmetric	Asymmetric	Antisymmetric
Relation R on set A is symmetric if $(x, y) \in R$ and $(y, x) \in R$	Relation R on a set A is asymmetric if $(x, y) \in R$, but $(y, x) \notin R$	Relation R of a set A is anti–symmetric if $(x, y) \in R$ and $(y, x) \in R$, then $x = y$
"Is equal to" is a symmetric relation, such as 6 = 4 + 2 and 2 + 4 = 6.	"Is greater than" is an asymmetric, such as 10 > 8 but 8 is not greater than 10	If $a \neq b$, then $(b, a) \in R$

(iv) Transitive Relation : Let A be any set. A relation R on set A is said to be a transitive relation *iff* $(x, y) \in R$ and $(y, z) \in R \Rightarrow (x, z) \in R$ for all $x, y, z \in A$ *i.e.,* xRy and $yRz \Rightarrow xRz$ for all $x, y, z \in A$. Transitivity fails only when there exists x, y, z such that $x\,Ry$, $y\,Rz$ but $x \not R z$.

Let's take some examples :

(A) The relation "is perpendicular to" in the set of straight lines is symmetry because if the line l is perpendicular to the line m, then m is also perpendicular to l.

(B) The relation "is more than" in the set of numbers is not symmetry because if a is greater than b, then b is not greater than a.

(C) Let A = {1, 2, 3} and R = {(1, 2); (2,1);(1, 3);(3,1)}. Then R is symmetry

- **For example,** Consider the set A = {1, 2, 3} and the relations
  $R_1 = \{(1, 2), (1, 3)\}$; $R_2 = \{(1, 2)\}$; $R_3 = \{(1, 1)\}$;
  $R_4 = \{(1, 2), (2, 1), (1, 1)\}$.
  Then R_1, R_2, R_3 are transitive while R_4 is not transitive since in R_4,$(2, 1) \in R_4$; $(1, 2) \in R_4$ but $(2, 2) \notin R_4$.
- The identity and the universal relations on a non-void sets are transitive.

(v) Equivalence Relation : A relation R on a set A is said to be an equivalence relation on A *iff*

(A) It is reflexive *i.e.* $(a, a) \in R$ for all $a \in A$

(B) It is symmetric *i.e.* $(a, b) \in R \Rightarrow (b, a) \in R$, for all a, b $\in A$

(C) It is transitive *i.e.* $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$ for all $a, b, c \in A$.

Congruence modulo (*m*) : Let m be an arbitrary but fixed integer. Two integers a and b are said to be congruence modulo m if $a - b$ is divisible by m and we write $a \equiv b \pmod{m}$.

Thus $a \equiv b \pmod{m} \Leftrightarrow a - b$ is divisible by m. For example, $18 \equiv 3 \pmod 5$ because $18 - 3 = 15$ which is divisible by 5. Similarly, $3 \equiv 13 \pmod 2$ because $3 - 13 = -10$ which is divisible by 2. But $25 \not\equiv 2 \pmod 4$ because 4 is not a divisor of $25 - 2 = 23$.

The relation "Congruence modulo *m*" is an equivalence relation.

- **Equivalence Classes of an Equivalence Relation**
  - Let R be equivalence relation in A ($\neq \phi$). Let $a \in A$. Then the equivalence class of a, denoted by $[a]$ or $\{\overline{a}\}$ is defined as the set of all those points of A which are related to a under the relation R. Thus $[a] = \{x \in A : x \, R \, a\}$.
  - It is easy to see that :
    (1) $b \in [a] \Rightarrow a \in [b]$
    (2) $b \in [a] \Rightarrow [a] = [b]$
    (3) Two equivalence classes are either disjoint or identical.

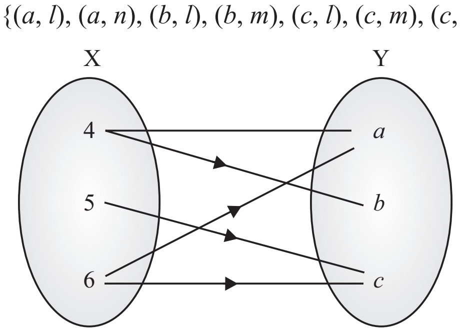
16. Composition of Relations

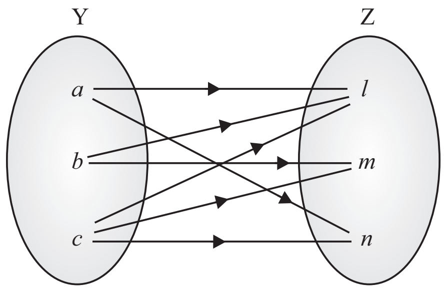
Let A, B, and C be sets, and let R be a relation from A to B and let S be a relation from B to C, *i.e.*, R is a subset of A × B and S is a subset of B × C, then R and S give rise to a relation from A to C indicated by R o S and defined by a (R o S) c if for some $b \in B$ we have aRb and bSc. *i.e.*, R o S = {(a, c)| there exists $b \in B$ for which $(a, b) \in R$ and $(b, c) \in S$}

For example, Let X = {4, 5, 6}, Y = {a, b, c} and Z = {l, m, n}. Consider the relation R_1 from X to Y and R_2 from Y to Z.

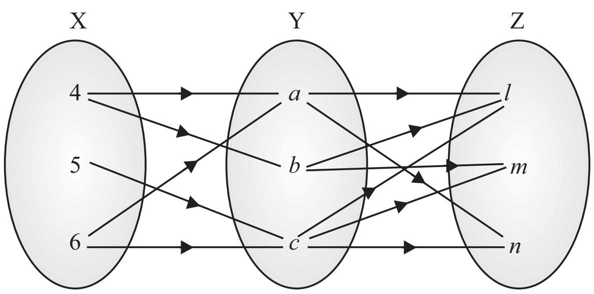
$R_1 = \{(4, a), (4, b), (5, c), (6, a), (6, c)\}$

$R_2 = \{(a, l), (a, n), (b, l), (b, m), (c, l), (c, m), (c, n)\}$





The composition relation R_1 o R_2 as shown in fig:



Composition of Relations

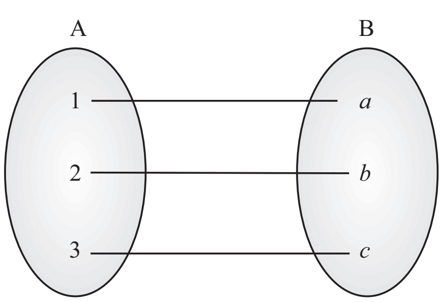
R_1 o R_2 = {(4, *l*), (4, *n*), (4, *m*), (5, *l*), (5, *m*), (5, *n*), (6, *l*), (6, *m*), (6, *n*)}

16.1 Properties of Composition of Relations

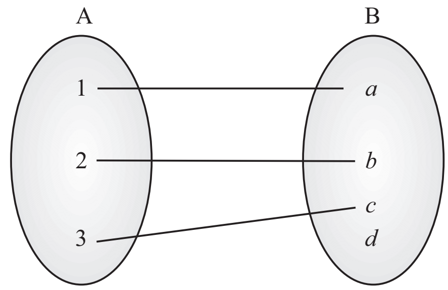
- The composition of the relation is not commutative *i.e.*, R o S ≠ S o R.
- The composition of relation is associative, *i.e.*, T o (S o R) = (T o S) o R.
- $(S \text{ o } R)^{-1} = R^{-1} \text{ o } S^{-1}$.

17. Functions

- Function is defined as a rule or a manner or a mapping or a correspondence *f* which maps each & every element of set A with a unique element of set B.
- Let A and B be any two non-empty sets, mapping from A to B will be a function only when every element in set A has one and only one image in set B.
- It is denoted by $f : A \rightarrow B$ or $A \xrightarrow{f} B$ and it can be read as "*f* is a function from A to B"



(a)



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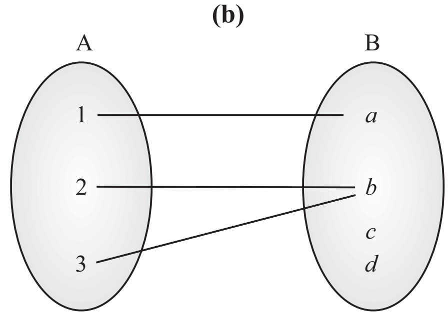

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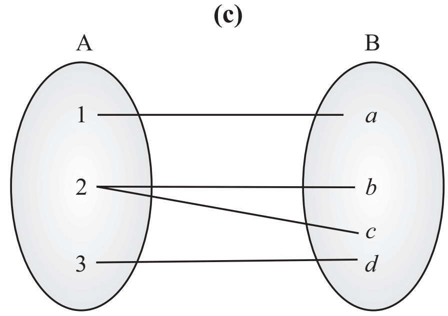


(d)

18. Real Function

A function whose range is a subset of real numbers is called real valued function.

Further, it domain of the function is either real number or a subset of real numbers, then it is called a real function.

Important

- Every real function is a real valued function but converse is not true *e.g.*, $f : C \rightarrow R$ is a real valued function but not a real function, where C is the set of complex numbers.

Note

- There may exist some elements in set B which are not the images of any element in set A.
- To each and every independent element in A, there corresponds one and only one image in B.
- The number of functions from a finite set A to finite set B $= [n(B)]^{[n(A)]}$

19. Algebra of Real Function

Let $f : X \rightarrow R$ and $g : X \rightarrow R$ be any two real functions, where $X \subseteq R$. Then, we define

(1) $(f \pm g) : X \rightarrow R$ by $(f \pm g)(x) = f(x) \pm g(x)$, for all $x \in X$.

(2) $(f.g) : X \rightarrow R$ by $(f.g)(x) = f(x).g(x)$, for all $x \in X$.

(3) $\left(\frac{f}{g}\right): X \to R$ by $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$ provided $g(x) \neq 0, x \in X$.

Let $f: X \to R$ and α be a real number. The, we define $(\alpha f): X \to R$ by $(\alpha f)(X) = \alpha f(x)$, for all $x \in X$.

Illustration

Let, $f(x) = x^2$ and $g(x) = 2x + 1$ be two real functions, Then,

(1) $(f + g)(x) = x^2 + 2x + 1$

(2) $(f - g)(x) = x^2 - 2x - 1$

(3) $(f.g)(x) = x^2 (2x + 1) = 2x^3 + x^2$

(4) $\left(\frac{f}{g}\right)(x) = \frac{x^2}{2x+1} . x \neq -\frac{1}{2}$

Example

Q. 1. Which of the following correspondences can be called a function ?

(A) $f(x) = x^3$; $\{-1, 0, 1\} \to \{0, 1, 2, 3\}$

(B) $f(x) = \pm\sqrt{x}$; $\{0, 1, 4\} \to \{-2, -1, 0, 1, 2\}$

(C) $f(x) = \sqrt{x}$; $\{0, 1, 4\} \to \{-2, -1, 0, 1, 2\}$

(D) None of the above

Sol. : (C) The function $f(x) = x^3$; $\{-1, 0, 1\} \to \{0, 1, 2, 3\}$ is not a function as $f(-1) = -1 \notin$ co-domain.

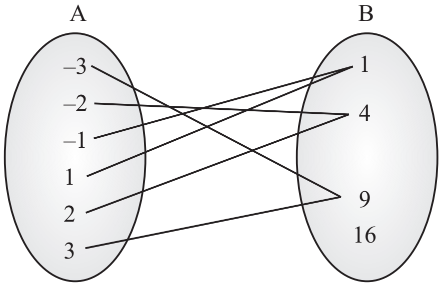
The function $f(x) = \pm\sqrt{x}$; $\{0, 1, 4\} \to \{-2, -1, 0, 1, 2\}$ is not a function as $f(4) = \pm 2$. Therefore, definition of function is not satisfied.

The function $f(x) = \sqrt{x}$; $\{0, 1, 4\} \to \{-2, -1, 0, 1, 2\}$ is a function as definition of function is satisfied.

20. Domain, Co-domain and Range of a Function

- Let $f: A \to B$ be a function, then the set A is known as the domain of f & the set B is known as co-domain of f. If a element 'a' of A is associated to the member 'b' of B, then 'b' is called the f-image of 'a' and we write $b = f(a)$. Further 'a' is called a pre-image of 'b'. The set $\{f(a): \forall\ a \in A\}$ is called the range of f and is denoted by $f(A)$. Clearly $f(A) \subseteq B$.

  For example, Let A = $\{-3, -2, -1, 1, 2, 3\}$, B = $\{1, 4, 9, 16\}$ and the function is. $f: A \xrightarrow{x^2} B$. The graphical representation of the function is



Here, Domain of the function is $\{-3, -2, -1, 1, 2, 3\}$, range of the function is $\{1, 4, 9\}$ and co-domain of the function is $\{1, 4, 9, 16\}$.

20.1 Methods for Finding Domain and Range of Function

The general formulas used to find the domain of different types of functions are :

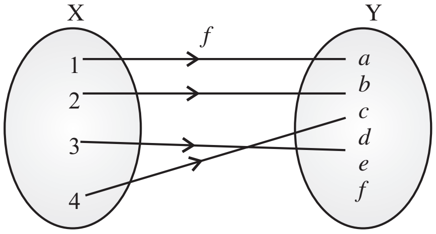
- Domain of any polynomial (linear, quadratic, cubic, etc) function is R.
- Domain of a square root function $\sqrt{x}$ is $x \geq 0$.
- Domain of an exponential function is R.
- Domain of logarithmic function is $x > 0$.
- To find the domain of a rational function $y = f(x)$, set the denominator $\neq 0$.
- If domain of $y = f(x)$ and $y = g(x)$ are D_1 and D_2 respectively then the domain of $f(x) \pm g(x)$ or $f(x).g(x)$ is $D_1 \cap D_2$.

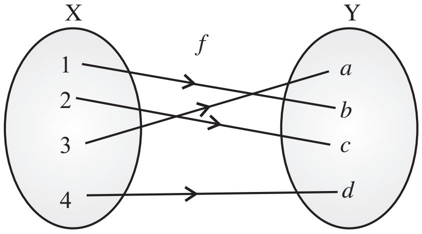
The general formulas used to find the range of different types of functions are :

- Range of any linear function is R.
- Range of a quadratic function is $y = a(x - h)^2 + k$ is $y \geq k$, if $a > 0$ and $y \leq k$, if $a < 0$.
- Range of a square root function $y = \sqrt{x}$ is $y \geq 0$.
- Range of an exponential function is $y \geq 0$.
- Range of logarithmic function is R.
- To find the range of a rational function $y = f(x)$, solve it for x and set the denominator $\neq 0$.

21. Types of Functions

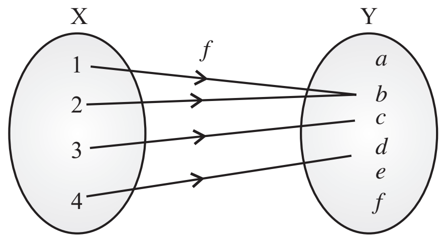
One-one or Injective Function : A function $f: X \to Y$ is defined to be *one-one (or injective),* if the images of distinct elements of X under f are distinct, *i.e.*, for every $x, y \in X$. $f(x) \neq f(y)$

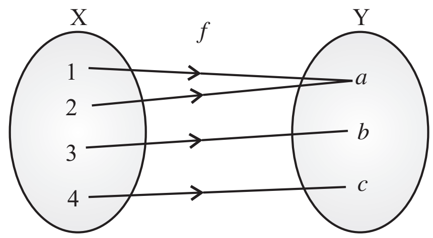




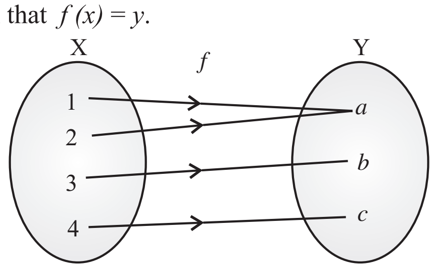
Many-one Function : A function $f: X \to Y$ is defined to be many-one function, if the two or more elements of X under f have the same image.

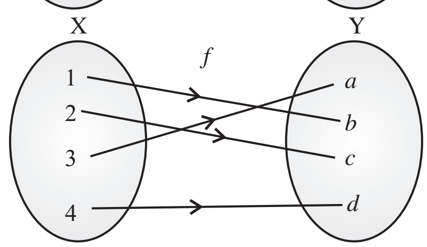
If $x, y \in$ X such that $x \neq y$, but $f(x) = f(y)$





Onto or Surjective Function : A function f : X $\rightarrow$ Y is said to be *onto* (or *surjective*), if every element of Y is the image of some element of X under f *i.e.*, for every $y \in$ Y, there exists an element x in X such that $f(x) = y$.

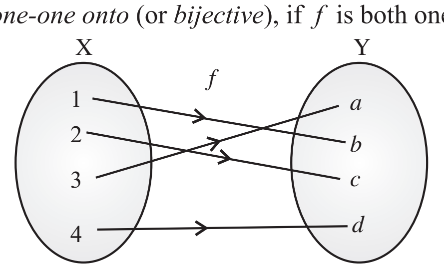




Note

f : X $\rightarrow$ Y is onto if and only if Range of f = Y

One-one onto or Bijective Function : A function f : X $\rightarrow$ Y is said to be *one-one onto* (or *bijective*), if f is both one-one onto.



22. No. of Functions

Note

Let f : A $\rightarrow$ B be a function, where n(A) = m and n(B) = n. Then,

(1) The number of function from A to B is the same as the number of ways of assigning images to every element of A and hence it is n^m.

(2) If n(B) < n(A), then there is no one–one function possible from A to B.

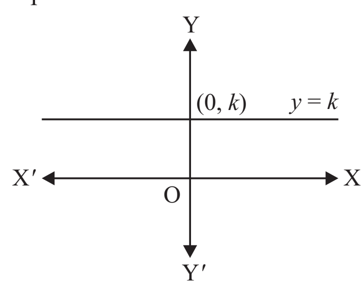
(3) If n(B) ≥ n(A), then number of one–one functions from A to B is ${}^{n}P_{m}$.

(4) The number of one–one function from A to A is m.

23. Classification of Functions

23.1 Constant Function

- A constant function is a function which does not change its parameters or range for different values of the domain. In other words, a constant function is a function that always gives or returns the same value.
- Graphically a constant function is a straight line, which is parallel to the x-axis. The domain of the function is the x-value and is represented on the x-axis, and the range of the function is y or $f(x)$ which is marked with reference to the y-axis.
- Let k be a constant, then function $f(x) = k \ \forall \ x \in$ R is known as constant function. The graphical representation of the function is



Here, Domain of $f(x)$ = R and Range of $f(x) = \{k\}$

23.2 Polynomial Function

- A polynomial function is a function that involves only non-negative integer powers or only positive integer exponents of a variable like the quadratic equation, cubic equation, etc.
- The function $y = f(x) = a_0x^n + a_1x^{n-1} + ... + a_n$, where a_0, $a_1, a_2, ..., a_n$ are real coefficients and n is a non-negative integer, is known as a polynomial function.
- There are various types of polynomial functions based on the degree of the polynomial. The most common types are :
  - **Zero Polynomial Function:** $P(x) = a_0x^0$
  - **Linear Polynomial Function:** $P(x) = a_0x + a_1$
  - **Quadratic Polynomial Function:** $P(x) = a_0x^2 + a_1x + a_2$
  - **Cubic Polynomial Function:** $P(x) = a_0x^3 + a_1x^2 + a_2x + a_3$

- **Quadratic Polynomial Function:** $P(x) = a_0x^4 + a_1x^3 + a_2x^2 + a_3x + a_4$

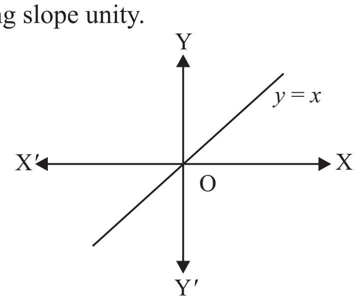
Note

All constant functions are linear functions.

- The Domain of $P(x) = R$ and Range of the polynomial function varies from function to function.

23.3 Identity Function

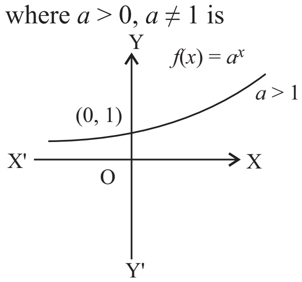
- An Identity function is a function where each element gives the image of itself as the same element.
- For the real valued function, the function $f : R \to R$ such that $f(a) = a\ \forall\ a \in R$ is known as Identity function.
- Identity function is a straight line passing through origin and having slope unity.



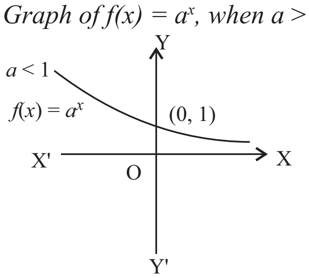
- Domain of $f(x) = R$ and Range of a function $= R$

23.4 Exponential Function

- An exponential function is a Mathematical function in form $f(x) = a^x$, where "x" is a variable and "a" is a constant which is called the base of the function and it should be greater than 0. The most commonly used exponential function base is the transcendental number, which is approximately equal to 2.71828.
- Let $a \neq 1$ be a positive real number, then $f : R \to (0, \infty)$ defined by $f(x) = a^x$ called exponential function.
- The graphical representation of the function $f(x) = a^x$, where $a > 0$, $a \neq 1$ is



Graph of $f(x) = a^x$, when $a > 1$

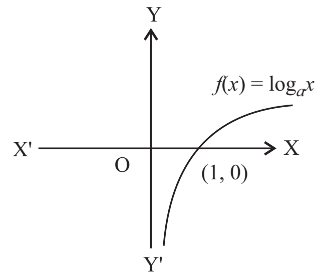


Graph of $f(x) = a^x$, when $a < 1$

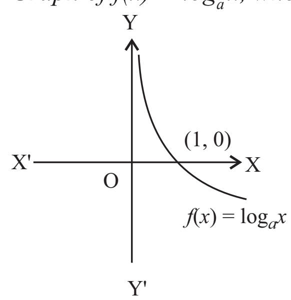
- The Domain of the function $f(x) = a^x$ is R and its range is $(0, \infty)$.

23.5 Logarithmic Function

- The logarithmic function is an inverse function of exponential function.
- Let $f : (0, \infty) \to R$ be a function defined by $f(x) = \log_a x$ such that $a \neq 1$, $a > 0$ and $x > 0$ is called logarithmic function.



Graph of $f(x) = \log_a x$, when $a > 1$

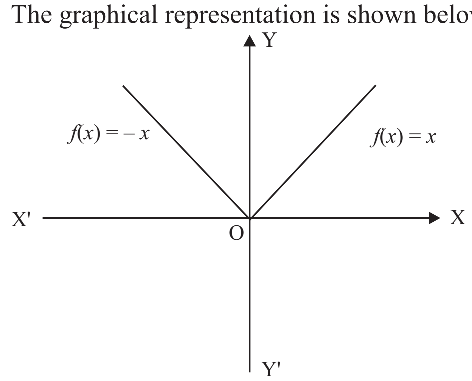


Graph of $f(x) = \log_a x$, when $a < 1$

- The domain of the function $f(x) = \log_a x$, where $x > 0$, $a > 0$ and $a \neq 1$ is $(0, \infty)$ and its range is R.

23.6 Modulus Function

- A modulus function is a function which gives the absolute value of a number or variable.
- The function defined by $f(x) = |x| = \begin{cases} x, \text{when } x \geq 0 \\ -x, \text{when } x < 0 \end{cases}$ is called the modulus function. The domain of the modulus function is the set R of all real numbers and the range is the set of all non-negative real numbers.
- The graphical representation is shown below :

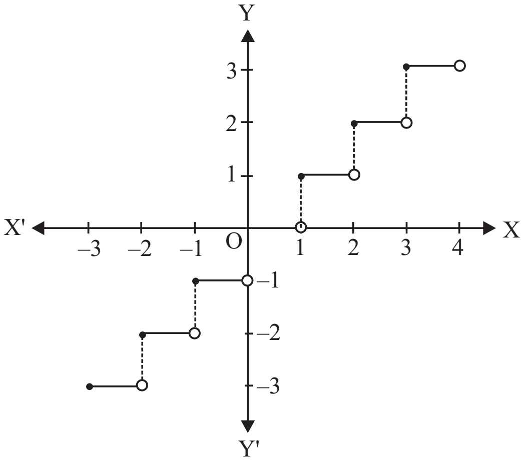


- The domain of the function $f(x) = |x|$ is R and its range is $R^+ \cup \{0\}$.

23.7 Greatest Integer Function

- Let $f(x) = [x]$, where $[x]$ denotes the greatest integer less than or equal to x.

- For example, [1.1] = 1, [2.2] = 2, [– 0.9] = –1, [– 2.1] = – 3 etc. The function f defined by $f(x) = [x]$ for all $x \in$ R, is called the greatest integer function.
- The graphical representation of the greater integer function is given below :



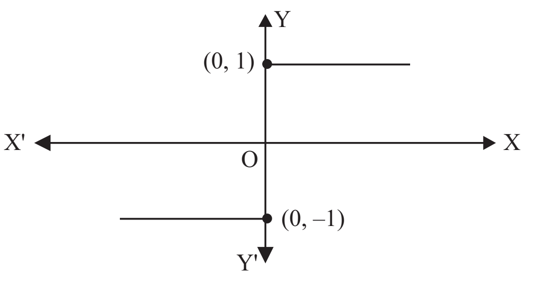
- The domain of the greater integer function is R and its range is I.

23.8 Signum Function

- Signum function helps determine the sign of the real value function, and attributes +1 for positive input values of the function, and attributes –1 for negative input values of the function.
- The function defined by $f(x) = \begin{cases} \dfrac{|x|}{x}, x \neq 0 \\ 0, x = 0 \end{cases}$

or $f(x) = \begin{cases} 1, & x = 0 \\ 0, & x = 0 \\ -1, & x < 0 \end{cases}$ is called the signum function.

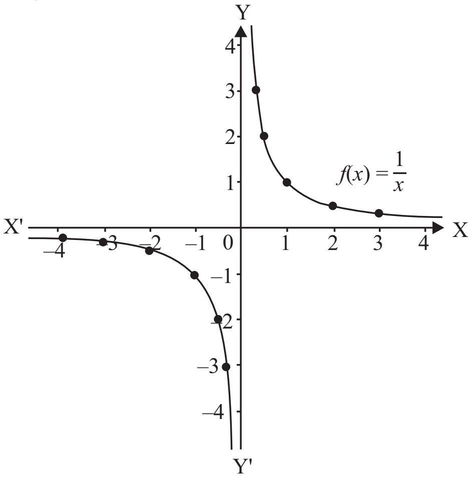
- The graphical representation of the signum function is given below :



- The domain of the signum function is R and its range is the set {–1, 0, 1}

23.9 Reciprocal Function

- A reciprocal function is obtained by finding the inverse of a given function. For a function $f(x) = x$, the reciprocal function is $f(x) = \dfrac{1}{x}$. The reciprocal function is also the multiplicative inverse of the given function.
- The graphical representation of the reciprocal function is given below :



- The domain and range of the reciprocal function are both equal to R – {0} *i.e.*, the set of all non-zero real numbers.

23.10 Explicit & Implicit Functions

- A function is said to be explicit function if its variable x, y can be easily separated.

  A function is said to be implicit function if the variable x, y cannot be separated easily.

23.11 Even and Odd Functions

- **Even Function :**
  - In a real valued function, after substituting $(-x)$ in place of x in the given function, we get the expression the same as the original *i.e.*, $f(-x) = f(x)$, $\forall\, x \in$, domain, then function $f(x)$ is called even function.
  - The graph of the even function is symmetric with respect to the Y-axis.
  - For example, $f(x) = e^x + e^{-x}$, $f(x) = x^2$, $f(x) = x \sin x$, $f(x) = \cos x$, $f(x) = x^2 \cos x$ are all are even functions.
- **Odd Functions :**
  - In a real valued function, after substituting $(-x)$ in place of x in the given function, we get the result *i.e.*, $f(-x) = -f(x)$, $\forall\, x \in$ domain, then function $f(x)$ is called odd function.
  - The graph of the odd function is symmetric with respect to the origin.
  - For example, $f(x) = e^x - e^{-x}$, $f(x) = \sin x$, $f(x) = x^3$, $f(x) = x \cos x$, $f(x) = x^2 \sin x$ all are odd functions.

23.12 Periodic Function

- A function is said to be periodic function if its each value is repeated after a particular interval. So a function $f(x)$ will be periodic if a positive real number T exists such that, $f(x + T) = f(x)$, $\forall\, x \in$ domain. Here the least positive value of T is called the period of the function.
- For example – The sine function *i.e.* $\sin x$ has a period of 2π because 2π is the smallest number for which $\sin(x + 2\pi) = \sin x$, for all x.

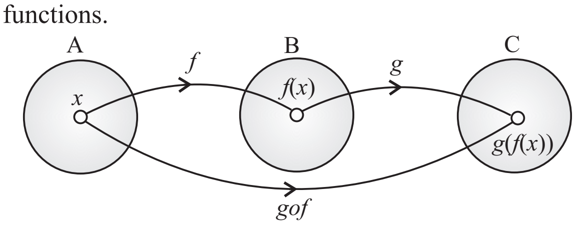
23.13 Inverse of Functions

- Let $f : X \rightarrow Y$ such that $f(x) = y$ be a one–one and onto function. Then, we can define a unique function $g : Y \rightarrow X$ such that $g(y) = x$, where $x \in X$ and $y = f(x)$. $Y \in Y$.
- Here, domain of g = range of f and range of g = domain of f, g is called the inverse of and is denoted by f^{-1}. Further, g is also one–one and onto and inverse of g is f.

  Thus, $g^{-1} = (f^{-1})^{-1} = f$.
- Similarly, $y = \sin^{-1} x$ is called the inverse of sine function and we read as sine inverse of x.

24. Composition of a Function

- Let $f : A \rightarrow B$ and $g : B \rightarrow C$ are two functions then the composite function of f and g, denoted by gof $A \rightarrow C$ will be defined as $gof(x) = g[f(x)]$, $\forall x \in A$.
- The below figure shows the representation of composite functions.



- gof exists iff the range of f is a subset of domain of g. And, fog exists if range of g is a subset of domain of.

24.1 Properties of Composition of Function

- The composition of the function is not commutative *i.e.*, $fog \neq gof$.
- The composition of the function is associative *i.e.*, $fo(goh) = (fog)\, oh$.
- If the functions f and g both are even functions, then the composition function fog is also an even function.
- If the functions f and g both are odd functions, then the composition function fog is also an odd function.
- If the function f is even and the function g is an odd function, then the composition function fog is an even function.
- If the function f is odd function and the function g is an even function, then the composition function fog is an even function.
- If $fog = gof$ then

  (i) either $f^{-1} = g$ or $g^{-1} = f$

  (ii) $(fog)(x) = (gof)(x) = (x)$

Examples

Q. 1. Domain and range of $f(x) = \dfrac{|x-3|}{x-3}$ are respectively :

(A) R, [–1, 1] (B) R – {3}, {1, –1}
(C) R^T, R (D) None of these

Sol. : (B) $f(x) = \dfrac{|x-3|}{x-3}$

$\Rightarrow \quad x - 3 \neq 0$

$\Rightarrow \quad x \neq 3$

Therefore, Domain is R – {3}

$$f(x) = \begin{cases} \dfrac{x-3}{x-3} = 1, & x > 3 \\ \dfrac{-(x-3)}{x-3} = -1, & x < 3 \end{cases}$$

Therefore, Range = {–1, 1}

Q. 2. The domain of the function $f(x) = \sqrt{(2 - 2x - x^2)}$ is :

(A) $-\sqrt{3} \le x \le \sqrt{3}$
(B) $-1-\sqrt{3} \le x \le -1+\sqrt{3}$
(C) $-2 \le x \le 2$
(D) $-2-\sqrt{3} \le x \le -2+\sqrt{3}$

Sol. : (B) $f(x) = \sqrt{2 - 2x - x^2}$

$\Rightarrow \quad 2 - 2x - x^2 > 0$

$\Rightarrow \quad x^2 + 2x - 2 < 0$

$\Rightarrow \quad x^2 + 2x + 1 - 3 < 0$

$\Rightarrow \quad (x + 1)^2 - (\sqrt{3})^2 < 0$

$-\sqrt{3} \le x+1 \le \sqrt{3}$

$\Rightarrow \quad -1-\sqrt{3} \le x \le -1+\sqrt{3}$

Q. 3. Let $f = \left\{\left(x, \dfrac{x^2}{1+x^2}\right) : x \in R\right\}$ be a function from R to R.

Determine the range of f.

(A) [0, 1) (B) [0, 1]
(C) [0, 2) (D) None of these

Sol. : (A) Let $y = \dfrac{x^2}{1+x^2}$

y is positive for all values of x

and $1 + x^2 > x^2$

$\Rightarrow \quad 1 > \dfrac{x^2}{1+x^2}$

$\Rightarrow \quad 1 > y$

Hence, $0 \le y < 1$

Range of $f = [0, 1)$

Q. 4. If $[x]^2 - 5[x] + 6 = 0$, where [.] denote the greatest integer function, then :

(A) $x \in [3, 4]$ (B) $x \in [2, 3]$
(C) $x \in [2, 3]$ (D) $x \in [2, 4)$

Sol. : (D) $[x]^2 - 5[x] + 6 = 0$

$[x]^2 - 3[x] - 2[x] + 6 = 0$

$$[x]([x]-3)-2([x]-3) = 0$$
$$([x]-2)([x]-3) = 0$$
$$[x] = 2 \text{ and } [x] = 3$$

If $[x] = 2$, then $x \in [2, 3)$

and if $[x] = 3$, then $x \in [3, 4)$

Hence, $x \in [2, 4)$

Q. 5. Given, $f(x) = \log\left(\frac{1+x}{1-x}\right)$ and $g(x) = \frac{3x+x^3}{1+3x^2}$, then $fog(x)$ equals :

(A) $-f(x)$ (B) $3f(x)$

(C) $[f(x)]^3$ (D) None of these

Sol. : (B) $fog(x) = f[g(x)]$

$$= f\left[\frac{3x+x^3}{1+3x^2}\right]$$

$$= \log\left(\frac{1+\left(\frac{3x+x^3}{1+3x^2}\right)}{1-\left(\frac{3x+x^3}{1+3x^2}\right)}\right)$$

$$= \log\left[\frac{1+3x^2+3x+x^3}{1+3x^2-3x-x^3}\right]$$

$$= \log\left(\frac{1+x}{1-x}\right)^3$$

$$= 3\log\left(\frac{1+x}{1-x}\right)$$

$$= 3f(x)$$

Q. 6. Let $f : [0, 1] \to [0, 1]$ define dy $f(x) = \frac{1-x}{1+x}$, $0 \le x \le 1$ and let $g : [0, 1] \to [0, 1]$ be defined by $g(x) = 4x(1-x)$, $0 \le x \le 1$, then fog and gof are :

(A) $\frac{(2x-1)^2}{1+4x-4x^2}, \frac{8x(1-x)}{(1+x)^2}$

(B) $\frac{(2x-1)}{1+4x-4x^2}, \frac{8(1-x)x}{(1+x)^2}$

(C) $\frac{(2x+1)^2}{1+4x+4x^2}, \frac{8}{(1+x)^2}$

(D) $\frac{(2x+1)^2}{(1+x)^2}, \frac{8(1-x)}{(1+x)^2}$

Sol. : (A) $fog = f[4x(1-x)]$

$$= \frac{1-4x(1-x)}{1+4x(1-x)}$$

$$= \frac{1-4x+4x^2}{1+4x-4x^2} = \frac{(2x-1)^2}{1+4x-4x^2}$$

$$gof = g\left[\frac{1-x}{1+x}\right]$$

$$= 4\left(\frac{1-x}{1+x}\right)\left[1-\left(\frac{1-x}{1+x}\right)\right]$$

$$= 4\left(\frac{1-x}{1+x}\right)\left[\frac{1+x-1+x}{1+x}\right]$$

$$= \frac{8x(1-x)}{(1+x)^2}$$

Important Questions

1. Consider the following statements :

1. The set of all irrational numbers between $\sqrt{12}$ and $\sqrt{15}$ is an infinite set.
2. The set of all odd integers less than 1000 is a finite set.

Which of the statement given above is/are correct :

(A) 1 only (B) 2 only

(C) Both 1 and 2 (D) Neither 1 nor 2

[NDA & NA 01-09-2024 (II)]

2. Let P and Q be two non-void relations on a set A. Which of the following statements are correct ?

1. P and Q are reflexive $\Rightarrow$ P $\cap$ Q is reflexive.
2. P and Q are symmetric $\Rightarrow$ P $\cup$ Q is symmetric.
3. P and Q are transitive $\Rightarrow$ P $\cap$ Q is transitive.

Select the answer using the code given below :

(A) 1 and 2 only (B) 2 and 3 only

(C) 1 and 3 only (D) 1, 2 and 3

[NDA & NA 01-09-2024 (II)]

3. If A and B are two non-empty sets having 10 elements in common, then how many elements do A × B and B × A have in common ?

(A) 10 (B) 20

(C) 40 (D) 100

[NDA & NA 01-09-2024 (II)]

4. In a class of 240 students, 180 passed in English, 130 passed in Hindi and 150 passed in Sanskrit. Further, 60 passed in only one subject, 110 passed in only two subjects and 10 passed in none of the subjects. How many passed in all three subjects ?

(A) 60 (B) 55

(C) 40 (D) 35

[NDA & NA 01-09-2024 (II)]

5. Let $z = [y]$ and $y = [x] - x$, where [.] is the greatest integer function. If x is not an integer but positive, then what is the value of z ?

(A) –1 (B) 0

(C) 1 (D) 2

[NDA & NA 01-09-2024 (II)]

6. If $f(x) = 4x + 1$ and $g(x) = kx + 2$ such that $fog(x) = gof(x)$, then what is the value of k ?

(A) 7 (B) 5

(C) 4 (D) 3

[NDA & NA 01-09-2024 (II)]

7. If $f(2x) = 4x^2 + 1$, then for how many real values of x will $f(2x)$ be the GM of $f(x)$ and $f(4x)$?

(A) Four (B) Two
(C) One (D) None

[NDA & NA 01-09-2024 (II)]

8. If $f(x) = [x]^2 - 30[x] + 221 = 0$, where $[x]$ is the greatest integer function, then what is the sum of all integer solutions.

(A) 13 (B) 17
(C) 27 (D) 30

[NDA & NA 01-09-2024 (II)]

9. If $f(x) = 9x - 8\sqrt{x}$ such that $g(x) = f(x) - 1$, then which one of the following is correct ?

(A) $g(x) = 0$ has no real roots
(B) $g(x) = 0$ has only one real root which is an integer.
(C) $g(x) = 0$ has two real roots which are integers
(D) $g(x) = 0$ has only one real root which is not an integer.

[NDA & NA 01-09-2024 (II)]

10. Let $f(x) f(y) = f(xy)$ for all real x, y, If $f(2) = 4$, then what is the value of $f(1/2)$?

(A) 1/4 (B) 1/2
(C) 1 (D) 4

[NDA & NA 01-09-2024 (II)]

Direction (Q. No. 11 and 12)

Consider the following for the two items that follow :

[NDA & NA 01-09-2024 (II)]

Let $fog(x) = \cos^2 \sqrt{x}$ and $gof(x) = |\cos x|$.

11. Which one of the following is $f(x)$?

(A) $\cos x$ (B) $\cos x^2$
(C) $\cos^2 x$ (D) $\cos |x|$

12. Which one of the following is $g(x)$?

(A) $\sqrt{x}$ (B) $|x|$
(C) x^2 (D) $x|x|$

13. Let A = $\{x \in$ R : $-1 < x < 1\}$. Which of the following is/are bijective functions from A to itself ?

1. $f(x) = x|x|$
2. $g(x) = \cos(\pi x)$

Select the correct answer using the code given below:

(A) 1 only (B) 2 only
(C) Both 1 and 2 (D) Neither 1 nor 2

[NDA & NA 2024 (I)]

14. Let R be a relation on the open interval $(-1, 1)$ and is given by

R = $\{(x, y) : |x + y| < 2\}$.

Then which one of the following is correct?

(A) R is reflexive but neither symmetric nor transitive
(B) R is reflexive and symmetric but not transitive
(C) R is reflexive and transitive but not symmetric
(D) R is an equivalence relation

[NDA & NA 2024 (I)]

15. For any three non-empty sets A, B, C, what is $(A \cup B) - \{(A - B) \cup (B - A) \cup (A \cap B)\}$ equal to?

(A) Null set
(B) A
(C) B
(D) $(A \cup B) - (A \cap B)$

[NDA & NA 2024 (I)]

16. Let A = $\{1, 2, 3, 4, 5\}$ and B = $\{6, 7\}$. What is the number of onto functions from A and B?

(A) 10 (B) 20
(C) 30 (D) 32

[NDA & NA 2024 (I)]

17. If $f(x) = ax - b$ and $g(x) = cx + d$ are such that $f[g(x)] = g[f(x)]$, then which one of the following holds?

(A) $f(d) = g(b)$ (B) $f(b) + g(d) = 0$
(C) $f(a) + g(c) = 2a$ (D) $f(d) + g(b) = 2d$

[NDA & NA 2024 (I)]

18. Which one of the following is correct in respect of $f(x) = \dfrac{1}{\sqrt{|x| - x}}$ and $g(x) = \dfrac{1}{\sqrt{x - |x|}}$?

(A) $f(x)$ has some domain and $g(x)$ has no domain
(B) $f(x)$ has no domain and $g(x)$ has some domain
(C) $f(x)$ and $g(x)$ have the same domain
(D) $f(x)$ and $g(x)$ do not have any domain

[NDA & NA 2024 (I)]

19. The Cartesian Product A × A has 16 elements among which are (0, 2) and (1, 3). Which of the following statements is/ are correct?

1. It is possible to determine set A.
2. A × A contains the element (3, 2).

Select the correct answer using the code given below :

(A) 1 only (B) 2 only
(C) Both 1 and 2 (D) Neither 1 nor 2

[NDA & NA 03-09-2023 (II)]

20. Let A = $\{1, 2, 3, ..., 20\}$. Define a relation R from A to A by R = $\{(x, y) : 4x - 3y = 1\}$, where $x, y \in$ A. Which of the following statements is/are correct?

1. The domain of R is $\{1, 4, 7, 10, 13, 16\}$.
2. The range of R is $\{1, 5, 9, 13, 17\}$.
3. The range of R is equal to codomain of R.

Select the correct answer using the code given below:

(A) 1 only (B) 2 only
(C) 1 and 2 (D) 2 and 3

[NDA & NA 03-09-2023 (II)]

21. Consider the following statements:

1. The relation f defined by
$$f(x) = \begin{cases} x^3, & 0 \le x \le 2 \\ 4x & 2 \le x \le 8 \end{cases}$$ is a function.

2. The relation g defined by
$$g(x) = \begin{cases} x^2, & 0 \le x \le 4 \\ 3x & 4 \le x \le 8 \end{cases}$$ is a function.

Which of the statements given above is/ are correct?

(A) 1 only (B) 2 only
(C) Both 1 and 2 (D) Neither 1 nor 2

[NDA & NA 03-09-2023 (II)]

22. Consider the following statements :

1. $A = (A \cup B) \cup (A - B)$
2. $A \cup (B - A) = (A \cup B)$
3. $B = (A \cup B) - (A - B)$

Which of the statements given above are correct?

(A) 1 and 2 only (B) 2 and 3 only
(C) 1 and 3 only (D) 1, 2 and 3

[NDA & NA 03-09-2023 (II)]

23. A function satisfies $f(x - y) = \dfrac{f(x)}{f(y)}$, where $f(y) \neq 0$. If $f(1) = 0.5$, then what is $f(2) + f(3) + f(4) + f(5) + f(6)$ equal to?

(A) $\dfrac{15}{32}$ (B) $\dfrac{17}{32}$
(C) $\dfrac{29}{64}$ (D) $\dfrac{31}{64}$

[NDA & NA 03-09-2023 (II)]

Direction (Q. No. 24 and 25)

Consider the following for the next questions that follow:

Let $f(x) = x^2 - 1$ and $gof(x) = x - \sqrt{x} + 1$.

24. Which one of the following is a possible expression for $g(x)$

(A) $\sqrt{x+1} - \sqrt[4]{x+1}$
(B) $\sqrt{x+1} - \sqrt[4]{x+1} + 1$
(C) $\sqrt{x+1} + \sqrt[4]{x+1}$
(D) $x + 1 - \sqrt{x+1} + 1$

[NDA & NA 03-09-2023 (II)]

25. What is $g(15)$ equal to?

(A) 1 (B) 2
(C) 3 (D) 4

[NDA & NA 03-09-2023 (II)]

Direction (Q. No. 26 and 27)

Consider the following for the next questions that follow:
A function is defined by $f(x) = \pi + \sin^2 x$

26. What is the range of the function?

(A) $[0, 1]$ (B) $[\pi, \pi + 1]$
(C) $[\pi - 1, \pi + 1]$ (D) $[\pi - 1, \pi - 1]$

[NDA & NA 03-09-2023 (II)]

27. What is the period of the function?

(A) 2π
(B) π
(C) $\frac{\pi}{2}$
(D) The function is non-periodic

[NDA & NA 03-09-2023 (II)]

Direction (Q. No. 28 and 29)

Consider the following for the next questions that follow:
Let $f(x) = \sqrt{2-x} + \sqrt{2+x}$

28. What is the domain of the function?

(A) $(-2, 2)$ (B) $[-2, 2]$
(C) $R - (-2, 2)$ (D) $R - [-2, 2]$

[NDA & NA 03-09-2023 (II)]

29. What is the greatest value of the function?

(A) $\sqrt{3}$ (B) $\sqrt{6}$
(C) $\sqrt{8}$ (D) 4

[NDA & NA 03-09-2023 (II)]

30. A mapping $f : A \to B$ defined as $f(x) = \frac{2x+3}{3x+5}$, $x \in A$. If f is to be onto, then what are A and B equal to?

(A) $A = \frac{R}{\left\{-\frac{5}{3}\right\}}$ and $B = \frac{R}{\left\{-\frac{2}{3}\right\}}$

(B) $A = R$ and $B = \frac{R}{\left\{-\frac{5}{3}\right\}}$

(C) $A = \frac{R}{\left\{-\frac{3}{2}\right\}}$ and $B = \frac{R}{\{0\}}$

(D) $A = \frac{R}{\left\{-\frac{5}{3}\right\}}$ and $B = \frac{R}{\left\{\frac{2}{3}\right\}}$

[NDA & NA 16-04-2023 (I)]

Direction (Q. No. 31 and 32)

Consider the following for the next two (02) items that follow:
Consider the function
$f(x) = |x - 2| + |3 - x| + |4 - x|$, where $x \in R$

31. At what value of x does the function attain minimum value?

(A) 2 (B) 3
(C) 4 (D) 0

[NDA & NA 16-04-2023 (I)]

32. What is the minimum value of the function?

(A) 2 (B) 3
(C) 4 (D) 0

[NDA & NA 16-04-2023 (I)]

Direction (Q. No. 33 and 34)

Consider the following for the next two (02) items that follow:
Let $f(x) = \sin[\pi^2]x + \cos[-\pi^2]x$ where [.] is a greatest integer function.

33. What is $f\left(\frac{\pi}{2}\right)$ equal to?

(A) -1 (B) 0
(C) 1 (D) 2

[NDA & NA 16-04-2023 (I)]

34. What is $f\left(\frac{\pi}{4}\right)$ equal to?

(A) $-\frac{1}{\sqrt{2}}$ (B) -1
(C) 1 (D) $\frac{1}{\sqrt{2}}$

[NDA & NA 16-04-2023 (I)]

35. If $f(x) = x^2 + 2$ and $g(x) = 2x - 3$, then what is $(fg)(1)$ equal to?

(A) 3 (B) 1
(C) -2 (D) -3

[NDA & NA 16-04-2023 (I)]

36. What is the range of the function $f(x) = x + |x|$ if the domain is the set of real numbers?

(A) $(0, \infty)$ (B) $[0, \infty)$
(C) $(-\infty, \infty)$ (D) $[1, \infty)$

[NDA & NA 16-04-2023 (I)]

37. If $f(x) = x(4x^2 - 3)$, then what is $f(\sin \theta)$ equal to?

(A) $-\sin 3\theta$ (B) $-\cos 3\theta$
(C) $\sin 3\theta$ (D) $-\sin 4\theta$

[NDA & NA 16-04-2023 (I)]

38. Consider the following statements :

1. If f is the subset of $Z \times Z$ defined by $f = \{(xy, x - y); x, y \in Z\}$, then f is a function from Z to Z.
2. If f is the subset of $N \times N$ defined by $f = \{(xy, x + y); x, y \in N\}$, then f is a function from N to N.

Which of the statements given above is/are correct ?

(A) Only 1 (B) Only 2
(C) Both 1 and 2 (D) Neither 1 nor 2

[NDA & NA 04-09-2022 (II)]

39. Consider the following statements :

1. The set of all irrational numbers between $\sqrt{2}$ and $\sqrt{5}$ is an infinite set.
2. The set of all odd integers less than 100 is a finite set.

Which of the statements given above is/are correct ?

(A) Only 1 (B) Only 2
(C) Both 1 and 2 (D) Neither 1 nor 2

[NDA & NA 04-09-2022 (II)]

40. Let A = {7, 8, 9, 10, 11, 12, 13, 14, 15, 16} and let $f : A \to N$ be defined by $f(x)$ = the highest prime factor of x. How many elements are there in the range of f ?

(A) 4 (B) 5
(C) 6 (D) 7

[NDA & NA 04-09-2022 (II)]

41. Let R be a relation from N to N defined by $R = \{(x, y) : x, y \in N \text{ and } x^2 = y^3\}$. Which of the following are not correct ?

1. $(x, x) \in R$ for all $x \in N$
2. $(x, y) \in R \Rightarrow (y, x) \in R$
3. $(x, y) \in R$ and $(y, z) \in R \Rightarrow (x, z) \in R$

Select the correct answer using the code given below :

(A) 1 and 2 only (B) 2 and 3 only
(C) 1 and 3 only (D) 1, 2 and 3

[NDA & NA 04-09-2022 (II)]

42. Consider the following :

1. $A \cap B = A \cap C \Rightarrow B = C$
2. $A \cup B = A \cup C \Rightarrow B = C$

Which of the above is/are correct ?

(A) Only 1 (B) Only 2
(C) Both 1 and 2 (D) Neither 1 nor 2

[NDA & NA 04-09-2022 (II)]

Direction (Q. No. 43 to 45)

Consider the following for the next three items that follow:
Let $f(x)$ be a function satisfying $f(x + y) = f(x) f(y)$ for all $x, y \in N$ such that $f(1) = 2$:

43. If $\sum_{x=2}^{n} f(x) = 2044$, then what is the value of n ?

(A) 8 (B) 9
(C) 10 (D) 11

[NDA & NA 04-09-2022 (II)]

44. What is $\sum_{x=1}^{5} f(2x-1)$ equal to ?

(A) 341 (B) 682

(C) 1023 (D) 1364

[NDA & NA 04-09-2022 (II)]

45. What is $\sum_{x=1}^{6} 2^x f(x)$ equal to ?

(A) 1365 (B) 2730

(C) 4024 (D) 5460

[NDA & NA 04-09-2022 (II)]

Direction (Q. No. 46 to 48)

Consider the following for the next three items that follow:
A university awarded medals in basket ball, football and volleyball. Only x students ($x < 6$) got medal in all the three sports and the medals went to a total of $15x$ students. It awarded $5x$ medals in basketball, $(4x + 15)$ medals in football and $(x + 25)$ medals in volleyball.

46. How many received medals in exactly two of the three sports ?

(A) $30 - 4x$ (B) $35 - 7x$

(C) $40 - 7x$ (D) $45 - 5x$

[NDA & NA 04-09-2022 (II)]

47. How many received medals in at least two of three sports ?

(A) $30 - 6x$ (B) $35 - 6x$

(C) $40 - 5x$ (D) $40 - 6x$

[NDA & NA 04-09-2022 (II)]

48. How many received medals in exactly one of three sports ?

(A) $21x - 40$ (B) $21x - 35$

(C) $20x - 35$ (D) $20x - 25$

[NDA & NA 04-09-2022 (II)]

49. If $f(x) = 4x + 1$ and $g(x) = kx + 2$ such that $fog(x) = gof(x)$, then what is the value of k ?

(A) 7 (B) 5

(C) 4 (D) 3

[NDA & NA 04-09-2022 (II)]

50. What is the minimum value of the function $f(x) = \log_{10}(x^2 + 2x + 11)$?

(A) 0 (B) 1

(C) 2 (D) 10

[NDA & NA 04-09-2022 (II)]

51. If A = {{1, 2, 3}}, then how many elements are there in the power set of A ?

(A) 1 (B) 2

(C) 4 (D) 8

[NDA & NA 10-04-2022 (I)]

52. Consider all the subsets of the set A = {1, 2, 3, 4}. How many of them are supersets of the set {4}?

(A) 6 (B) 7

(C) 8 (D) 9

[NDA & NA 10-04-2022 (I)]

53. Consider the following statements in respect of two non-empty sets A and B :

1. $x \notin (A \cup B) \Rightarrow x \notin A$ or $x \notin B$

2. $x \notin (A \cap B) \Rightarrow x \notin A$ and $x \notin B$

Which of the above statements is/are correct ?

(A) 1 only (B) 2 only

(C) Both 1 and 2 (D) Neither 1 nor 2

[NDA & NA 10-04-2022 (I)]

54. Consider the following statements in respect of two non-empty set A and B :

1. $A \cup B = A \cap B$ iff $A = B$

2. $A \Delta B = \phi$ iff $A = B$

Which of the above statements is/are correct ?

(A) 1 only (B) 2 only

(C) Both 1 and 2 (D) Neither 1 nor 2

[NDA & NA 10-04-2022 (I)]

55. Consider the following statements in respect of the relation R in the set I N of natural numbers defined by xRy if $x^2 - 5xy + 4y^2 = 0$:

(1) R is reflexive

(2) R is symmetric

(3) R is transitive

Which of the above statements is/are correct ?

(A) 1 only (B) 2 only

(C) 1 and 2 only (D) 1, 2 and 3

[NDA & NA 10-04-2022 (I)]

56. Consider the following statements in respect of any relation R on a set A :

1. If R is reflexive, then R^{-1} is also reflexive

2. If R is symmetric, then R^{-1} is also symmetric

3. If R is transitive, then R^{-1} is also transitive

Which of the above statements are correct?

(A) 1 and 2 only (B) 2 and 3 only

(C) 1 and 3 only (D) 1, 2 and 3

[NDA & NA 10-04-2022 (I)]

57. What is the domain of the function $f(x) = \sqrt{1-(x-1)^2}$?

(A) (0, 1) (B) [–1, 1]

(C) (0, 2) (D) [0, 2]

[NDA & NA 10-04-2022 (I)]

58. If $4f(x) - f\left(\frac{1}{x}\right) = \left(2x + \frac{1}{x}\right)\left(2x - \frac{1}{x}\right)$, then what is $f(2)$ equal to ?

(A) 0 (B) 1

(C) 2 (D) 4

[NDA & NA 10-04-2022 (I)]

59. If $f(x) = 4x + 3$, then what is $f^{0}f^{0}f(-1)$ equal to ?

(A) –1 (B) 0

(C) 1 (D) 2

[NDA & NA 10-04-2022 (I)]

60. Consider the following statements in respect of sets :

1. The union over intersection of sets is distributive.

2. The complement of union of two sets is equal to intersection of their complements.

3. If the difference of two sets is equal to empty set, then the two sets must be equal.

Which of the above statements are correct?

(A) 1 and 2 only (B) 2 and 3 only

(C) 1 and 3 only (D) 1, 2 and 3

[NDA & NA (II) Dec. 2021]

61. Consider three sets X, Y and Z having 6, 5 and 4 elements respectively. All these 15 elements are distinct. Let S = (X – Y) $\cup$ Z. How many proper subsets does S have?

(A) 255 (B) 256

(C) 1023 (D) 1024

[NDA & NA (II) Dec. 2021]

62. Consider the following statements in respect of relations and functions :

1. All relations are functions but all functions are not relations.

2. A relation from A to B is a subset of Cartesian product A × B.

3. A relation in A is a subset of Cartesian product A × A.

Which of the above statements are correct?

(A) 1 and 2 only (B) 2 and 3 only

(C) 1 and 3 only (D) 1, 2 and 3

[NDA & NA (II) Dec. 2021]

63. Suppose set A consists of first 250 natural numbers that are multiples of 3 and set B consists of first 200 even natural numbers. How many elements doses A $\cup$ B have?

(A) 324 (B) 364

(C) 384 (D) 400

[NDA & NA (II) Dec. 2021]

64. What is the range of the function $f(x) = 1 - \sin x$ defined on entire real line?

(A) $(0, 2)$ (B) $[0, 2]$
(C) $(-1, 1)$ (D) $[-1, 1]$

[NDA & NA (II) Dec. 2021]

65. Consider the following statements :

1. $A = \{1, 3, 5\}$ and $B = \{2, 4, 7\}$ are equivalent sets.
2. $A = \{1, 5, 9\}$ and $B = \{1, 5, 5, 9, 9\}$ are equal sets.

which of the above statements is/are correct?

(A) 1 only (B) 2 only
(C) Both 1 and 2 (D) Neither 1 nor 2

[NDA & NA 2021 (I)]

66. Consider the following statement :

1. The null set is a subset of every set.
2. Every set is a subset of itself.
3. If a set has 10 elements, then its power set will have 1024 elements.

Which of the above statements are correct?

(A) 1 and 2 only
(B) 2 and 3 only
(C) 1 and 3 only
(D) 1, 2 and 3

[NDA & NA 2021 (I)]

67. Let R be a relation defined as $x\mathrm{R}y$ if and only if $2x + 3y = 20$, where $x, y \in \mathrm{N}$. How many elements of the form (x, y) are there in R?

(A) 2 (B) 3
(C) 4 (D) 6

[NDA & NA 2021 (I)]

68. Consider the following statement:

1. A function $f : \mathrm{Z} \to \mathrm{Z}$, defined by $f(x) = x + 1$, is one-one as well as onto.
2. A function $f : \mathrm{N} \to \mathrm{N}$, defined by $f(x) = x + 1$, is one-one but not onto.

Which of the above statements is/are correct?

(A) 1 only (B) 2 only
(C) Both 1 and 2 (D) Neither 1 nor 2

[NDA & NA 2021 (I)]

69. What is the domain of the function $f(x) = 3^x$?

(A) $(-\infty, \infty)$ (B) $(\infty, 0)$
(C) $[0, \infty)$ (D) $(-\infty,\infty) - \{0\}$

[NDA & NA 2021 (I)]

70. If $1.5 \le x \le 4.5$, then which one of the following is correct ?

(A) $(2x - 3)(2x - 9) > 0$
(B) $(2x - 3)(2x - 9) < 0$
(C) $(2x - 3)(2x - 9) \ge 0$
(D) $(2x - 3)(2x - 9) \le 0$

[NDA & NA 2020 (II)]

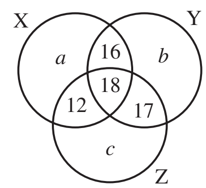
71. Let $S = \{1, 2, 3, ...\}$. A relation R on $S \times S$ is defined by $x\mathrm{R}y$ if $\log_a x > \log_a y$ when $a = \frac{1}{2}$. Then the relation is :

(A) reflexive only
(B) symmetric only
(C) transitive only
(D) both symmetric and transitive

[NDA & NA 2020 (II)]

Direction (Q. No. 72 to 74)

Consider the following Venn diagram, where X, Y and Z are three sets. Let the number of elements in Z be denoted by $n(\mathrm{Z})$ which is equal to 90.



[NDA & NA 2020 (II)]

72. If the number of elements in Y and Z are in the ratio 4 : 5, then what is the value of b ?

(A) 18 (B) 19
(C) 21 (D) 23

73. What is the value of $n(\mathrm{X}) + n(\mathrm{Y}) + n(\mathrm{Z}) - n(\mathrm{X} \cap \mathrm{Y}) - n(\mathrm{Y} \cap \mathrm{Z}) - n(\mathrm{X} \cap \mathrm{Z}) + n(\mathrm{X} \cap \mathrm{Y} \cap \mathrm{Z})$?

(A) $a + b + 43$ (B) $a + b + 63$
(C) $a + b + 96$ (D) $a + b + 106$

74. If the number of elements belonging to neither X, nor Y, Z is equal to p, then what is the number of elements in the complement of X ?

(A) $p + b + 60$ (B) $p + b + 40$
(C) $p + a + 60$ (D) $p + a + 40$

75. Consider the proper subsets of $\{1, 2, 3, 4\}$. How many of these proper subsets are superset of the set $\{3\}$?

(A) 5 (B) 6
(C) 7 (D) 8

[NDA & NA 2020 (II)]

76. What is the domain of the function $f(x) = \cos^{-1}(x - 2)$?

(A) $[-1, 1]$ (B) $[1, 3]$
(C) $[0, 5]$ (D) $[-2, 1]$

[NDA & NA 2020 (II)]

77. If $f(x) = 2x - x^2$, then what is the value of $f(x + 2) + f(x - 2)$ when $x = 0$?

(A) -8 (B) -4
(C) 8 (D) 4

[NDA & NA 2020 (II)]

78. Which one of the following is correct in respect of the graph of $y = \frac{1}{x-1}$?

(A) The domain is $\{x \in \mathrm{R} \mid x \ne 1\}$ and the range is the set of reals.
(B) The domain is $\{x \in \mathrm{R} \mid x \ne 1\}$, the range is $\{y \in \mathrm{R} \mid y \ne 0\}$ and the graph intersects y-axis at $(0, -1)$
(C) The domain is the set of reals and the range is the singleton set $\{0\}$
(D) The domain is $\{x \in \mathrm{R} \mid x \ne 1\}$ and the range is the set of points on the y-axis.

[NDA & NA 2020 (II)]

79. Suppose $X = \{1, 2, 3, 4\}$ and R is a relation on X. If $R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1), (2, 3), (3, 2)\}$, then which one of the following is correct ?

(A) R is reflexive and symmetric, but not transitive
(B) R is symemtric and transitive, but not reflexive
(C) R is reflexive and transitive, but not symmetric
(D) R is neither reflexive nor transitive, but symmetric

[NDA & NA 2019 (I)]

80. A relation R is defined on the set N of natural numbers as $x\mathrm{R}y \Rightarrow x^2 - 4xy + 3y^2 = 0$. Then which one of the following is correct ?

(A) R is reflexive and symmetric, but not transitive
(B) R is reflexive and transitive, but not symmetric
(C) R is reflexive, symmetric and transitive
(D) R is reflexive, but neither symmetric nor transitive

[NDA & NA 2019 (I)]

81. Consider the following statements for the two non-empty sets A and B :

1. $(\mathrm{A} \cap \mathrm{B}) \cup (\mathrm{A} \cap \overline{\mathrm{B}}) \cup (\overline{\mathrm{A}} \cap \mathrm{B}) = \mathrm{A} \cup \mathrm{B}$
2. $(\mathrm{A} \cup (\overline{\mathrm{A}} \cap \overline{\mathrm{B}})) = \mathrm{A} \cup \mathrm{B}$

Which of the above statements is/are correct ?

(A) only 1 (B) 2 only
(C) Both 1 and 2 (D) Neither 1 nor 2

[NDA & NA 2019 (I)]

82. Let X be a non-empty set and let A, B, C be subsets of X. Consider the following statements :

1. $\mathrm{A} \subset \mathrm{C} \Rightarrow (\mathrm{A} \cap \mathrm{B}) \subset (\mathrm{C} \cap \mathrm{B})$, $(\mathrm{A} \cup \mathrm{B}) \subset (\mathrm{C} \cup \mathrm{B})$

2. $(A \cap B) \subset (C \cap B)$ for all sets $B \Rightarrow A \subset C$
3. $(A \cup B) \subset (C \cup B)$ for all sets $B \Rightarrow A \subset C$

Which of the following statements are correct ?

(A) 1 and 2 only (B) 2 and 3 only
(C) 1 and 3 only (D) 1, 2 and 3

[NDA & NA 2019 (I)]

83. A function f defined by $f(x) = \ln(\sqrt{x^2+1} - x)$ is :

(A) an even function
(B) an odd function
(C) Both even and odd function
(D) Neither even nor odd function

[NDA & NA 2019 (I)]

84. Let $A \cup B = \{x|(x-a)(x-b) > 0$, where $a < b\}$. What are A and B equal to ?

(A) $A = \{x|x > a\}$ and $B = \{x|x > b\}$
(B) $A = \{x|x < a\}$ and $B = \{x|x > b\}$
(C) $A = \{x|x < a\}$ and $B = \{x|x < b\}$
(D) $A = \{x|x > a\}$ and $B = \{x|x < b\}$

[NDA & NA 2019 (II)]

Direction (Q. No. 85 and 86)

Read the following information and answer the two items that follow:

Let $f(x) = x^2$, $g(x) = \tan x$ and $h(x) = \ln x$.

[NDA & NA 2019 (II)]

85. For $x = \frac{\sqrt{\pi}}{2}$, what is the value of $[ho(gof)](x)$?

(A) 0 (B) 1
(C) $\frac{\pi}{4}$ (D) $\frac{\pi}{2}$

86. What is $[fo(fof)](2)$ equal to ?

(A) 2 (B) 8
(C) 16 (D) 256

87. Let A and B be subsets of X and $C = (A \cap B') \cup (A' \cap B)$, where A' and B' are complements of A and B respectively in X. What is C equal to?

(A) $(A \cup B') - (A \cap B')$
(B) $(A' \cup B) - (A' \cap B)$
(C) $(A \cup B) - (A \cap B)$
(D) $(A' \cup B') - (A' \cap B')$

[NDA & NA 2018 (I)]

Direction (Q. No. 88 and 89)

A survey was conducted among 300 students. It was found that 125 students like to play cricket, 145 students like to play football and 90 students like to play tennis. 32 students like to play exactly two games out of the three games.

[NDA & NA 2018 (II)]

88. How many students like to play all the three games ?

(A) 14 (B) 21
(C) 28 (D) 35

89. How many students like to play exactly only one game ?

(A) 196 (B) 228
(C) 254 (D) 268

90. Let S be the set of all persons living in Delhi. We say that x, y in S are related if they were born in Delhi on the same day. Which one of the following is correct ?

(A) The relation is an equivalent relation
(B) The relation is not reflexive but it is symmetric and transitive
(C) The relation is not symmetric but it is reflexive and transitive
(D) The relation is not transitive but it is reflexive and symmetric

[NDA & NA 2017 (I)]

91. Let $f : [-6, 6] \to R$ be defined by $f(x) = x^2 - 3$. Consider the following :

1. $(f \circ f \circ f)(-1) = (f \circ f \circ f)(1)$
2. $(f \circ f \circ f)(-1) - 4(f \circ f \circ f)(1) = (f \circ f)(0)$

Which of the above is/are correct ?

(A) 1 only (B) 2 only
(C) Both 1 and 2 (D) Neither 1 nor 2

[NDA & NA 2017 (I)]

92. Suppose there is a relation * between the positive numbers x and y given by $x * y$, if and only if $x \le y^2$. Then which one of the following is correct ?

(A) * is reflexive but not transitive and symmetric
(B) * is transitive but not reflexive and symmetric
(C) * is symmetric and reflexive but not transitive
(D) * is symmetric but not reflexive and transitive

[NDA & NA 2016 (I)]

93. If

$$f(x_1) - f(x_2) = f\left(\frac{x_1 - x_2}{1 - x_1 x_2}\right)$$

for $x_1, x_2 \in (-1, 1)$, then what is $f(x)$ equal to ?

(A) $\ln\left(\frac{1-x}{1+x}\right)$ (B) $\ln\left(\frac{2+x}{1-x}\right)$
(C) $\tan^{-1}\left(\frac{1-x}{1+x}\right)$ (D) $\tan^{-1}\left(\frac{1+x}{1-x}\right)$

[NDA & NA 2016 (I)]

Direction (Q. No. 94 and 95)

Consider the function $f(x) = \frac{27(x^{2/3} - x)}{4}$.

[NDA & NA 2016 (I)]

94. How many solutions does the function $f(x) = 1$ have ?

(A) One (B) Two
(C) Three (D) Four

95. How many solutions does the function $f(x) = -1$ have ?

(A) One (B) Two
(C) Three (D) Four

96. Let R be a relation on the set N of natural numbers defied by 'nR$m \Leftrightarrow n$, is a factor of m'. Then which one of the following is correct?

(A) R is reflexive, symmetric but not transitive
(B) R is transitive, symmetric but not reflexive
(C) R is reflexive, transitive but not symmetric
(D) R is an equivalence relation

[NDA & NA 2016 (I)]

97. Let S be a set of all distinct numbers of the form $\frac{p}{q}$, where $p, q \in \{1, 2, 3, 4, 5, 6\}$. What is the cardinality of the set S ?

(A) 21 (B) 23
(C) 32 (D) 36

[NDA & NA 2016 (II)]

98. Let X be the set of all persons living in a city. Persons x, y in X are said to be related as $x < y$ if y is at least 5years older than x. Which one of the following is correct ?

(A) The relation is an equivalence relation on X
(B) The relation is transitive but neither reflexive nor symmetric
(C) The relation is reflexive but neither transitive nor symmetric
(D) The relation is symmetric but neither transitive nor reflexive

[NDA & NA 2015 (I)]

99. In a class of 60 students, 45 students like music, 50 students like dancing, 5 students like neither. Then the number of students in the class who like both music and dancing is :

(A) 35 (B) 40
(C) 50 (D) 55

[NDA & NA 2015 (I)]

100. Let Z be the set of integers and aRb, where $a, b \in Z$ if and only if $(a - b)$, 5 is divisible by 5.

Consider the following statements :

1. The relation R partitions Z into five equivalent classes.
2. Any two equivalent classes are either equal or disjoint.

Which of the above statements is/are correct ?

(A) 1 only (B) 2 only
(C) Both 1 and 2 (D) Neither 1 nor 2

[NDA & NA 2015 (I)]

101. Let A = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}. Then the number of subsets of A containing exactly two elements is :

(A) 20 (B) 40
(C) 45 (D) 90

[NDA & NA 2015 (I)]

102. Let X be the set of all persons living in Delhi. The persons *a* and *b* in X are said to be related if the difference in their ages is at most 5 years. The relation is :

(A) an equivalence relation
(B) reflexive and transitive but not symmetric
(C) symmetric and transitive but not reflexive
(D) reflexive and symmetric but not transitive

[NDA & NA 2015 (II)]

103. If A = $\{x \in R : x^2 + 6x - 7 < 0\}$ and B = $\{x \in R : x^2 + 9x + 14 > 0\}$, then which of the following is/are correct ?

1. $(A \cap B) = (-2, 1)$
2. $(A \backslash B) = (-7, -2)$

Select the correct answer using the code given below :

(A) 1 only (B) 2 only
(C) Both 1 and 2 (D) Neither 1 nor 2

[NDA & NA 2015 (II)]

104. If $g(x) = \dfrac{1}{f(x)}$ and $f(x) = x, x \neq 0$, then which one of the following is correct ?

(A) $f(f(f(g(g(f(x)))))) = g(g(f(g(f(x)))))$
(B) $f(f(g(g(g(f(x)))))) = g(g(f(g(f(x)))))$
(C) $f(g(f(g(g(f(x)))))) = g(g(f(g(f(x)))))$
(D) $f(f(f(g(g(f(x)))))) = f(f(f(g(f(x)))))$

[NDA & NA 2015 (II)]

105. Let X be the set of all citizens of India. Elements *x, y* in X are said to be related if the difference of their age is 5 years. Which one of the following is correct ?

(A) The relation is an equivalence relation on X.
(B) The relation is symmetric but neither reflexive nor transitive.
(C) The relation is reflexive but neither symmetric nor transitive.
(D) None of the above

[NDA & NA 2014 (I)]

106. Consider the following relations from A to B where A = {*u, v, w, x, y, z*} and B = {*p, q, r, s*}

1. {(*u, p*), (*v, p*), (*w, p*), (*x, q*), (*y, q*), (*z, q*)}
2. {(*u, p*), (*v, p*), (*w, r*), (*z, s*)}
3. {(*u, s*), (*v, r*), (*w, q*), (*u, p*), (*v, q*), (*z, q*)}
4. {(*u, q*), (*v, p*), (*w, s*), (*x, r*), (*y, q*), (*z, s*)}

Which of the above relations are not functions ?

(A) 1 and 2 (B) 1 and 4
(C) 2 and 3 (D) 3 and 4

[NDA & NA 2014 (I)]

107. Let N denote the set of all non-negative integers and Z denote the set of all integers. The function $f : Z \to N$ given by $f(x) = |x|$ is :

(A) One-one but not onto
(B) Onto but not one-one
(C) Both one-one and onto
(D) Neither one-one nor onto

[NDA & NA 2014 (I)]

108. The function $f : N \to N$, N being the set of natural numbers, defined by $f(x) = 2x + 3$ is :

(A) injective and surjective
(B) injective but not surjective
(C) not injective but surjective
(D) neither injective nor surjective

[NDA & NA 2014 (II)]

Direction (Q. No. 109 to 111)

Consider the function $f(x) = \dfrac{x-1}{x+1}$.

[NDA & NA 2014 (II)]

109. What is $\dfrac{f(x)+1}{f(x)-1} + x$ equal to ?

(A) 0 (B) 1
(C) $2x$ (D) $4x$

110. What is $f(2x)$ equal to ?

(A) $\dfrac{f(x)+1}{f(x)+3}$ (B) $\dfrac{f(x)+1}{3f(x)+1}$

(C) $\dfrac{3f(x)+1}{f(x)+3}$ (D) $\dfrac{f(x)+3}{3f(x)+1}$

111. What is $f(f(x))$ equal to ?

(A) x
(B) $-x$
(C) $-\dfrac{1}{x}$
(D) None of the above

[NDA & NA 2014 (II)]

Direction (Q. No. 112 to 117)

In a state with a population of 75×10^6, 45% of them know Hindi, 22% know English, 18% know Sanskrit, 12% know Hindi and English, 8% know English and Sanskrit, 10% know Hindi and Sanskrit and 5% know all the three languages.

112. What is the number of people who do not know any of the above three languages ?

(A) 3×10^6 (B) 4×10^6
(C) 3×10^7 (D) 4×10^7

[NDA and NA Solved Paper 2013 (I)]

113. What is the number of people who know Hindi only ?

(A) 21×10^6 (B) 25×10^6
(C) 28×10^6 (D) 3×10^7

[NDA and NA Solved Paper 2013 (I)]

114. What is the number of people who know Sanskrit only ?

(A) 5×10^6
(B) 45×10^6
(C) 4×10^6
(D) None of the above

[NDA and NA Solved Paper 2013 (I)]

115. What is the number of people who know English only ?

(A) 5×10^6
(B) 45×10^5
(C) 4×10^6
(D) None of the above

[NDA and NA Solved Paper 2013 (I)]

116. What is the number of people who know only one language ?

(A) 3×10^6 (B) 4×10^6
(C) 3×10^7 (D) 4×10^7

[NDA and NA Solved Paper 2013 (I)]

117. What is the number of people who know only two languages ?

(A) 11.25×10^5 (B) 11.25×10^6
(C) 12×10^5 (D) 12.5×10^5

[NDA and NA Solved Paper 2013 (I)]

118. Universal set,

U = $\{x \mid x^5 - 6x^4 + 11x^3 - 6x^2 = 0\}$
A = $\{x \mid x^2 - 5x + 6 = 0\}$
B = $\{x \mid x^2 - 3x + 2 = 0\}$

What is $(A \cap B)'$ equal to?

(A) {1, 3} (B) {1, 2, 3}
(C) {0, 1, 3} (D) {0, 1, 2, 3}

[NDA/NA Math 2006-(I)]

119. Let A = $\{(n, 2n) : n \in N\}$ and B = $\{(2n, 3n) : n \in N\}$. What is $A \cap B$ equal to?

(A) $\{(n, 6n) : n \in N\}$

(B) $\{(2n, 6n) : n \in N\}$

(C) $\{(n, 3n) : n \in N\}$

(D) ϕ

[NDA/NA Math 2006-(I)]

120. Which one of the following sets has all elements as odd positive integers?

(A) $S = \{x \in R | x^3 - 8x^2 + 19x - 12 = 0\}$

(B) $S = \{x \in R | x^3 - 9x^2 + 23x - 15 = 0\}$

(C) $S = \{x \in R | x^3 - 7x^2 + 14x - 8 = 0\}$

(D) $S = \{x \in R | x^3 - 12x^2 + 44x - 48 = 0\}$

[NDA/NA Math 2006-(I)]

121. Let X be any non-empty set containing n elements. Then what is the number of relations on X?

(A) 2^{n^2} (B) 2^n

(C) 2^{2n} (D) n^2

[NDA/NA Math 2006-(I)]

122. Let X and Y be two non-empty sets and let R_1 and R_2 be two relations from X into Y. Then, which of the following is correct?

(A) $(R_1 \cap R_2)^{-1} \subset R_1^{-1} \cap R_2^{-1}$

(B) $(R_1 \cap R_2)^{-1} \supset R_1^{-1} \cap R_2^{-1}$

(C) $(R_1 \cap R_2)^{-1} = R_1^{-1} \cap R_2^{-1}$

(D) $(R_1 \cap R_2)^{-1} = R_1^{-1} \cap R_2^{-1}$

[NDA/NA Math 2006-(I)]

123. Let $x > y$ be two real numbers and $z \in R$, $z \neq 0$. Consider the following:

(i) $x + z > y + z$ and $xz > yz$

(ii) $x + z > y - z$ and $x - z > y - z$

(iii) $xz > yz$ and $\frac{x}{z} > \frac{y}{z}$

(iv) $x - z > y - z$ and $\frac{x}{z} > \frac{y}{z}$

Which of the above is/are correct?

(A) (i) only

(B) (ii) only

(C) (i) and (ii) only

(D) (i), (ii), (iii) and (iv)

[NDA/NA Math 2006-(I)]

124. Let $P = \{p_1, p_2, p_3, p_4\}$ $Q = \{q_1, q_2, q_3, q_4\}$ and $R = \{r_1, r_2, r_3, r_4\}$. If $S_{10} = \{(p_i, q_j, r_k) : i + j + k = 10\}$, how many elements does S_{10} have?

(A) 2 (B) 4

(C) 6 (D) 8

[NDA/NA Math 2006-(I)]

125. If F(n) denotes the set of all divisors of n except 1, what is the least value of y satisfying $[F(20) \cap F(16)] \subseteq F(y)$?

(A) 1 (B) 2

(C) 4 (D) 8

[NDA/NA Math 2006-(II)]

126. Let $f : [-100\pi, 100\pi] \rightarrow [-1, 1]$ be defined by $f(\theta) = \sin\theta$. Then what is the number of values of $\theta \in [-100\pi, 1000\pi]$ for which $f(\theta) = 0$?

(A) 1000 (B) 1101

(C) 1100 (D) 1110

[NDA/NA Math 2007-(I)]

127. A relation R is defined over the set of non-negative integers as $xRy \Rightarrow x^2 + y^2 = 36$ what is R?

(A) $\{(0, 6)\}$

(B) $\{(6,0),(\sqrt{11},5),(3,3,\sqrt{3})\}$

(C) $\{(6, 0), (0, 6)\}$

(D) $(\sqrt{11},5),(2,4\sqrt{2}),(5\sqrt{11}),(4\sqrt{2},2)$

[NDA/NA Math 2007-(I)]

128. If P(A) denotes the power set of A and A is the void set, then what is number of elements in P{P{P{(A)}}}?

(A) 0 (B) 1

(C) 4 (D) 16

[NDA/NA Math 2009-(II)]

129. If $N_a = \{ax \mid x \in N\}$, then what is $N_{12} \cap N_8$ equal to?

(A) N_{12} (B) N_{20}

(C) N_{24} (D) N_{48}

[NDA/NA Math 2009-(II)]

130. If $A = P(\{1, 2\})$ where P denotes the power set, then which one of the following is correct?

(A) $\{1, 2\} \subset A$ (B) $1 \in A$

(C) $\phi \notin A$ (D) $\{1, 2\} \in A$

[NDA/NA Math 2010-(I)]

131. If the cardinality of a set A is 4 and that of a set B is 3, then what is the cardinality of the set $A \Delta B$?

(A) 1

(B) 5

(C) 7

(D) Cannot be determined as the sets A and B are not given

[NDA/NA Math 2010-(II)]

132. If P, Q and R are three non-collinear points, then what is $PQ \cap PR$ equal to?

(A) Null set (B) {P}

(C) {P, Q, R} (D) {Q, R}

[NDA/NA Math 2011-(I)]

Answer Key

1. (A)	**2.** (A)	**3.** (D)	**4.** (A)	**5.** (A)
6. (A)	**7.** (C)	**8.** (D)	**9.** (B)	**10.** (A)
11. (C)	**12.** (A)	**13.** (A)	**14.** (D)	**15.** (A)
16. (C)	**17.** (D)	**18.** (A)	**19.** (C)	**20.** (C)
21. (A)	**22.** (B)	**23.** (D)	**24.** (B)	**25.** (C)
26. (B)	**27.** (B)	**28.** (B)	**29.** (C)	**30.** (D)
31. (B)	**32.** (A)	**33.** (B)	**34.** (D)	**35.** (D)
36. (B)	**37.** (A)	**38.** (D)	**39.** (A)	**40.** (C)
41. (D)	**42.** (D)	**43.** (C)	**44.** (B)	**45.** (D)
46. (C)	**47.** (D)	**48.** (A)	**49.** (A)	**50.** (B)
51. (B)	**52.** (C)	**53.** (D)	**54.** (C)	**55.** (A)
56. (D)	**57.** (D)	**58.** (D)	**59.** (A)	**60.** (A)
61. (C)	**62.** (B)	**63.** (C)	**64.** (B)	**65.** (C)
66. (D)	**67.** (B)	**68.** (C)	**69.** (A)	**70.** (D)
71. (C)	**72.** (C)	**73.** (D)	**74.** (A)	**75.** (C)
76. (B)	**77.** (A)	**78.** (B)	**79.** (D)	**80.** (D)
81. (A)	**82.** (D)	**83.** (B)	**84.** (B)	**85.** (A)
86. (D)	**87.** (C)	**88.** (A)	**89.** (C)	**90.** (A)
91. (C)	**92.** (A)	**93.** (A)	**94.** (B)	**95.** (A)
96. (C)	**97.** (B)	**98.** (B)	**99.** (B)	**100.** (C)
101. (C)	**102.** (D)	**103.** (C)	**104.** (B)	**105.** (B)
106. (C)	**107.** (B)	**108.** (B)	**109.** (A)	**110.** (C)
111. (C)	**112.** (C)	**113.** (A)	**114.** (D)	**115.** (D)
116. (C)	**117.** (B)	**118.** (C)	**119.** (D)	**120.** (B)
121. (A)	**122.** (D)	**123.** (D)	**124.** (C)	**125.** (B)
126. (B)	**127.** (C)	**128.** (D)	**129.** (C)	**130.** (D)
131. (D)	**132.** (B)			